

Two Way Beam Supported Slab Part 2

The following example was done by Mr. Naim Hassan, 3rd Year 2nd Semester Student of CE Dept., AUST

Given a slab of 16 feet by 14 feet and supported on all four edges by beams width of 12 inches on all four sides, beams' depth 12 inches below the slab. The slab is a typical interior slab.
 $f_y = 60 \text{ ksi}$, $f'_c = 3 \text{ ksi}$;

Solution:

$$h_f = \frac{(16+14) \times 12 \times 2}{160} = 4.5 \text{ inch}$$

$$\text{now, } \alpha_f = \frac{E_{cb} I_b}{E_{cs} I_s}$$

$$I_{S1,3} = \frac{16 \times 12 \times 4.5^3}{12} = 1458 \text{ in}^4$$

$$I_{S2,4} = \frac{14 \times 12 \times 4.5^3}{12} = 1275.75 \text{ in}^4$$

$$b_w + 2h_w \leq b_w + 8h_f \rightarrow 12 + 2 \times 12 \leq 12 + 8 \times 4.5$$

$$b_w + 2h_w = 36 \text{ in}$$

for beam the centroid is,

$$y = \frac{36 \times 4.5 \times \frac{4.5}{2} + 12 \times 12 \times (4.5 + \frac{12}{2})}{36 \times 4.5 + 12 \times 12} = 6.13 \text{ in}$$

$$I_b = \frac{12 \times 12^3}{12} + 12 \times 12 \times ((4.5 + 12/2) - 6.13)^2 + \frac{36 \times 4.5^3}{12} + 36 \times 4.5 \times (4.5/2 - 6.13)^2 = 7190.14 \text{ in}^4$$

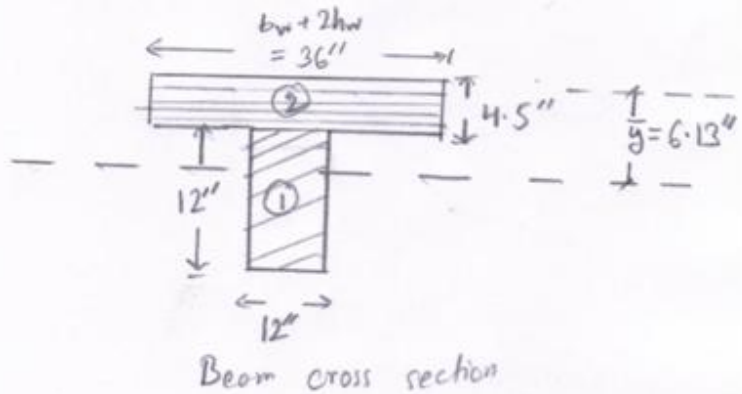
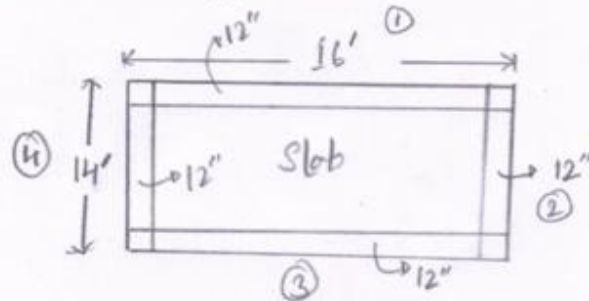
$$\alpha_{m1,3} = \frac{I_b}{I_{S1,3}} = \frac{7190.14}{1458} = 4.93$$

$$\alpha_{m2,4} = \frac{I_b}{I_{S2,4}} = \frac{7190.14}{1275.75} = 5.64$$

$$\alpha_{\text{mavg}} = \frac{4.93 + 5.64 + 4.93 + 5.64}{4} = 5.29$$

$$\text{Since } \alpha_{\text{mavg}} > 2 ; h_{\min} = \frac{l_n (0.8 + \frac{f_y}{200000})}{36 + 9\beta} = \frac{14 \times 12 (0.8 + \frac{60000}{200000})}{36 + 9 \times \frac{14}{12}} = 3.97 \text{ inch} = 4.0 \text{ inch}$$

$$h_{\min} = 4.0 \text{ inch}$$



The following Example was done by Md. Mahmudun Nobe, ID - 12.01.03.078, AUST Batch no. 28

Determination of minimum thickness of a slab

A two-way reinforced concrete building floor system is composed of slab panels measuring 20x25 ft in plan, supported by shallow column-line beams cast monolithically with the slab as shown in Fig. 02. Using concrete with $f'_c=4000$ psi and steel with $f_y= 60,000$ psi, determine the minimum thickness of the slab.

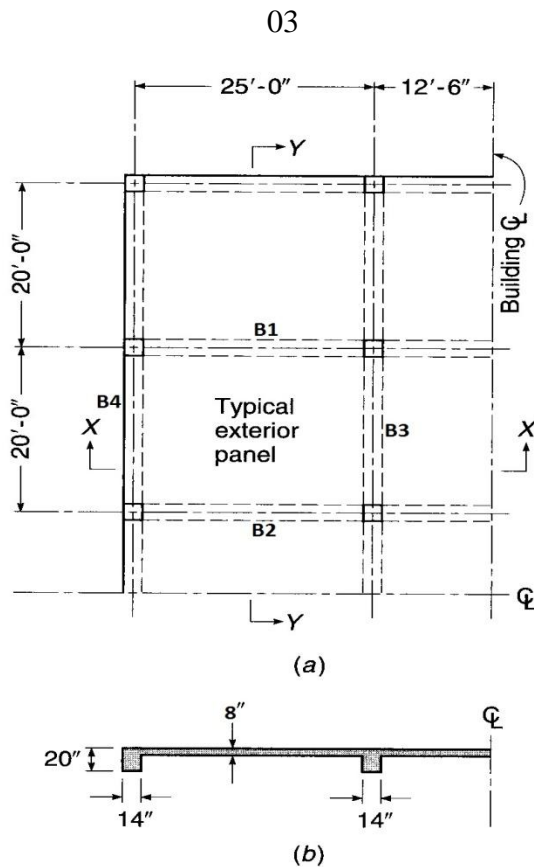


Figure 02: Two-way slab floor with beams on column lines:
 (a) Partial floor plan;
 (b) Section X-X (section Y-Y similar).

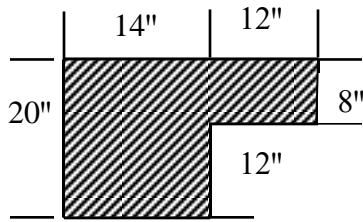
Solution: At first select the largest slab panel from floor slab plan. In this example, dimension of the slab panel is 20'×25'. Primarily, now we determining thickness of slab using the following formula:

$$\text{thickness (in)} = \frac{\text{Perimeter}}{145}$$

Here, Perimeter = $2 \times (20 + 25) \times 12 = 1080$ in

So thickness = $\frac{1080}{145} = 7.45$ in ≈ 8 in (say)

Moment of Inertia for beam B4 (Exterior beam):

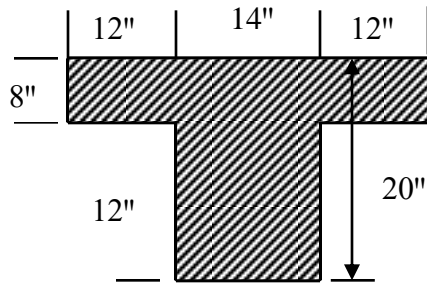


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$$\bar{Y}_4 = \frac{(20 \times 14 \times 10) + (12 \times 8 \times 4)}{(20 \times 14) + (12 \times 8)} = 8.47 \text{ in}$$

$$I_{b4} = \frac{14 \times 20^3}{12} + (20 \times 14) \times (10 - 8.47)^2 + \frac{12 \times 8^3}{12} + (12 \times 8) \times (8.47 - 4)^2 = 12418.95 \text{ in}^4$$

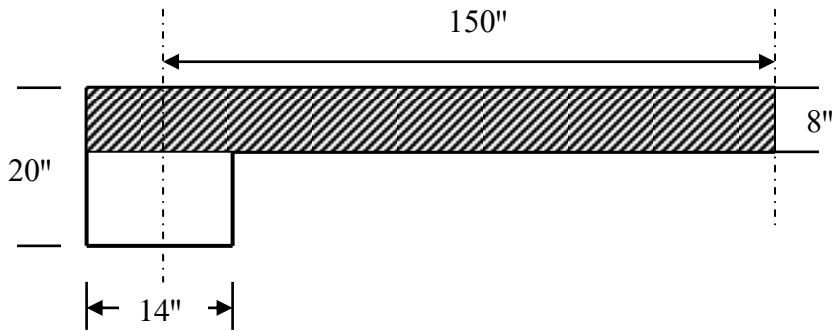
Moment of Inertia for Beam B1, B2, and B3 (Interior beam): In this case, those three beam's dimension is same and those are interior beam. So here moment of inertia for beam B1, B2, B3 is same.



$$\bar{Y}_1 = \bar{Y}_2 = \bar{Y}_3 = \frac{(20 \times 14 \times 10) + (12 \times 8 \times 4) \times 2}{(20 \times 14) + (12 \times 8) \times 2} = 7.56 \text{ in}$$

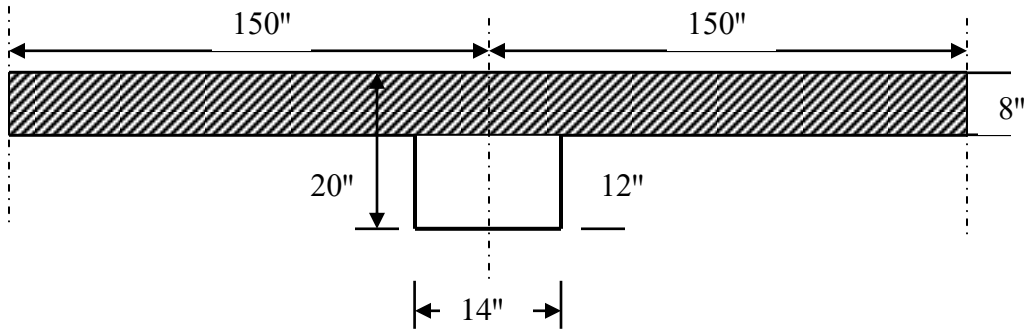
$$I_{b1} = I_{b2} = I_{b3} = \frac{14 \times 20^3}{12} + (20 \times 14) \times (10 - 7.56)^2 + \left(\frac{12 \times 8^3}{12} + (12 \times 8) \times (7.56 - 4)^2 \right) \times 2 = 14457.67 \text{ in}^4$$

Calculation of I_{s4} :



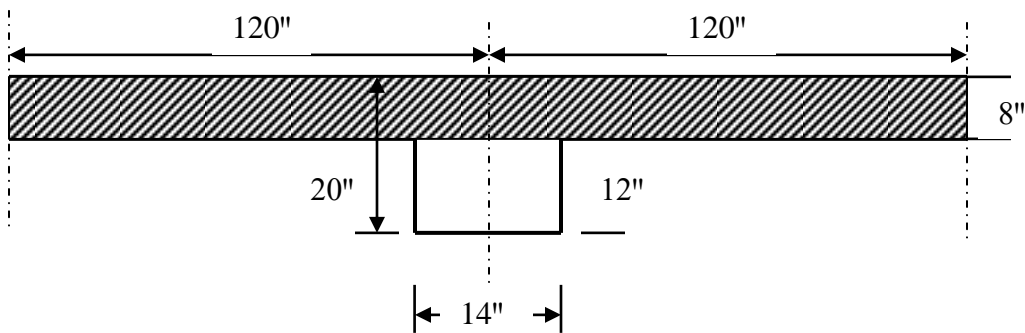
$$I_{s4} = \frac{(150+7) \times 8^3}{12} = 6698.67 \text{ in}^4$$

Calculation of I_{s3} :



$$I_{s3} = \frac{(150+150) \times 8^3}{12} = 12800 \text{ in}^4$$

Calculation of I_{s1} and I_{s2} : Value of I_{s1} and I_{s2} is same. Because B1 and B2 both are interior beam and for both cases, clear span on both side transverse to the beam B1 and B2 are same.



$$I_{s1} = I_{s2} = \frac{(120+120) \times 8^3}{12} = 10240 \text{ in}^4$$

Calculation of α : We know $\alpha = \frac{E_{cb} I_b}{E_{cs} I_s}$. Here $E_{cb} = E_{cs}$. Because of beam and slab concrete is same. So we can write $\alpha = \frac{I_b}{I_s}$.

$$\text{For this example } \alpha_1 = \frac{I_{b1}}{I_{s1}} = \frac{14457.67}{10240} = 1.41$$

$$\alpha_2 = \frac{I_{b2}}{I_{s2}} = \frac{14457.67}{10240} = 1.41$$

$$\alpha_3 = \frac{I_{b3}}{I_{s3}} = \frac{14457.67}{12800} = 1.13$$

$$\alpha_4 = \frac{I_{b4}}{I_{s4}} = \frac{12418.95}{6698.67} = 1.85$$

$$\text{Average value of } \alpha, \alpha_{avg} = \frac{1.41+1.41+1.13+1.85}{4} = 1.45$$

The ratio of long to short clear spans is $\beta = \frac{286}{226} = 1.27$. Then the minimum thickness is not to be less than that given by Eq. (13.8a):

$$h = \frac{286(0.8 + (\frac{60000}{200,000}))}{36 + 5 \times 1.27(1.45 - 0.2)} = 7.16 \text{ in}$$

The following Example was done by Md. Mahmudun Nobe, ID - 12.01.03.078, AUST Batch no. 28
Determination of minimum thickness of a slab

Design of two-way edge supported slab by using moment coefficients.

Beam-column supported floor slab of a 93'-6"×75'-6" (center to center distance of extreme columns) "cyclone shelter" is to carry service live load of 100 psf in addition to its own weight, 1/2" thick plaster and 3/2" thick floor finish. Supporting columns of 14 in square are spaced orthogonally at an interval at 31'-2" and 25'-2" on centers along longitudinal and transverse directions respectively. Width of each beam is 14 in. Using BNBC/ACI code of moment coefficients design the slab by USD method, if $f'_c = 3000$ psi and $f_y = 60000$ psi.

Solution:

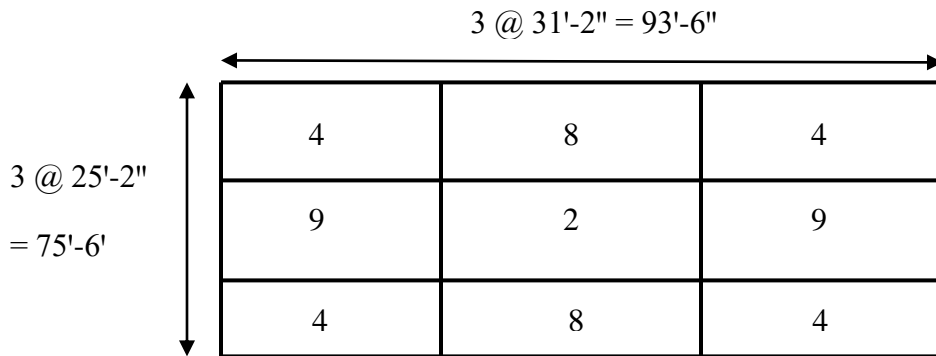


Figure 03: Slab panel orientation and case type, e.g., case 9 is typical exterior, 4 is corner slab etc.

Here $A = 25'2'' - 1'2'' = 24'$ and $B = 31'2'' - 1'2'' = 30' = l_n$.

$$t = \frac{l_n \left(0.8 + \left(\frac{f_y}{200000} \right) \right)}{36 + 9\beta} = \frac{30 \left(0.8 + \left(\frac{60000}{200000} \right) \right)}{36 + 9 \cdot \frac{30}{24}} = 8.38'' \approx 8.5'' \text{ say.}$$

So $d = 8.5'' - 1'' = 7.5'$

$$W_{DL} = (8.5 + 0.5 + 1.5) \cdot 12.5 \cdot 1.2 = 157.5 \text{ psf}$$

$$W_{LL} = 100 \cdot 1.6 = 160 \text{ psf}$$

$$\overline{W_u} = 317.5 \text{ psf}$$

$$m = A/B = 24/30 = 0.8$$

	2	4	8	9
$-C_A$	0.065	0.071	0.055	0.075
$-C_B$	0.027	0.029	0.041	0.017
$C_{A DL}$	0.026	0.039	0.032	0.029
$C_{B DL}$	0.011	0.016	0.015	0.010
$C_{A LL}$	0.041	0.048	0.044	0.042
$C_{B LL}$	0.017	0.020	0.019	0.017

○ Controlling coefficient.

[Note: In this slab, there are four different types of cases among all panels. We take the maximum value of moment coefficient from four cases.]

$$\begin{aligned}
 +M_A &= C_{A DL} * W_{DL} * A^2 + C_{A LL} * W_{LL} * A^2 \\
 &= 0.039 * 157.5 * 24^2 + 0.048 * 160 * 24^2 \\
 &= 7961.76 \text{ lb-ft/ft} \\
 &= 7.69 \text{ k-ft/ft}
 \end{aligned}$$

$$\begin{aligned}
 -M_A &= C_A * W_u * A^2 \\
 &= 0.075 * 317.5 * 24^2 \\
 &= 13716 \text{ lb-ft/ft} \\
 &= 13.6 \text{ k-ft/ft}
 \end{aligned}$$

$$\begin{aligned}
 +M_B &= C_{B DL} * W_{DL} * B^2 + C_{B LL} * W_{LL} * B^2 \\
 &= 0.016 * 157.5 * 30^2 + 0.020 * 160 * 30^2 \\
 &= 5148 \text{ lb-ft/ft} \\
 &= 5.148 \text{ k-ft/ft}
 \end{aligned}$$

$$\begin{aligned}
 -M_B &= C_B * W_u * B^2 \\
 &= 0.041 * 317.5 * 30^2 \\
 &= 11716 \text{ lb-ft/ft} \\
 &= 11.716 \text{ k-ft/ft}
 \end{aligned}$$

Rebar for short direction/transverse direction:

$$+A_s = \frac{M * 12}{0.9 * f_y * \left(d - \frac{a}{2}\right)} = \frac{M * 12}{0.9 * 60 * \left(d - \frac{a}{2}\right)} = \frac{M}{4.5 * \left(d - \frac{a}{2}\right)} = \frac{7.96}{4.5 * (7.5 - 0.24)} = 0.244 \text{ in}^2/\text{ft}$$

$$\text{and } a = \frac{A_s * f_y}{0.85 * f_c * b} = \frac{A_s * 60}{0.85 * 3 * 12} = 1.96 * A_s = 1.96 * 0.244 = 0.478 \text{ in.}$$

$$A_{\min} = 0.0018 * b * t * 1.5 = 0.0018 * 12 * 8.5 * 1.5 = 0.275 \text{ in}^2/\text{ft} \text{ (Controlling)}.$$

Using $\phi 10$ bar

$$S = \frac{\text{dia of bar used} * \text{width of strip}}{\text{required } A_s} = \frac{0.121 * 12}{0.275} = 5.28'' \approx 5'' \text{ c/c at bottom along short direction}$$

crank 50% bar to negative zone.

$$-A_{S A} = \frac{M}{4.5 * \left(d - \frac{a}{2}\right)} = \frac{13.61}{4.5 * (7.5 - 0.42)} = 0.427 \text{ in}^2/\text{ft} \text{ (Controlling).}$$

$$a = 1.96 * A_s = 0.838 \text{ in}$$

$$A_{\min} = 0.275 \text{ in}^2/\text{ft}.$$

$$\text{Already provided } A_{s1} = \frac{0.121 * 12}{10} = 0.1452 \text{ in}^2/\text{ft}$$

$$\text{Extra top required, } A_{s2} = (0.4275 - 0.1452) = 0.2823 \text{ in}^2/\text{ft}.$$

Using $\Phi 10$ bar $S = 5.14" \approx 5" \text{ c/c}$ extra top.

Rebar along long direction:

$$+A_{S B} = \frac{5.148}{4.5 * (7.5 - 0.15)} = 0.155 \text{ in}^2/\text{ft}$$

$$A_{\min} = 0.275 \text{ in}^2/\text{ft} \text{ (Controlling).}$$

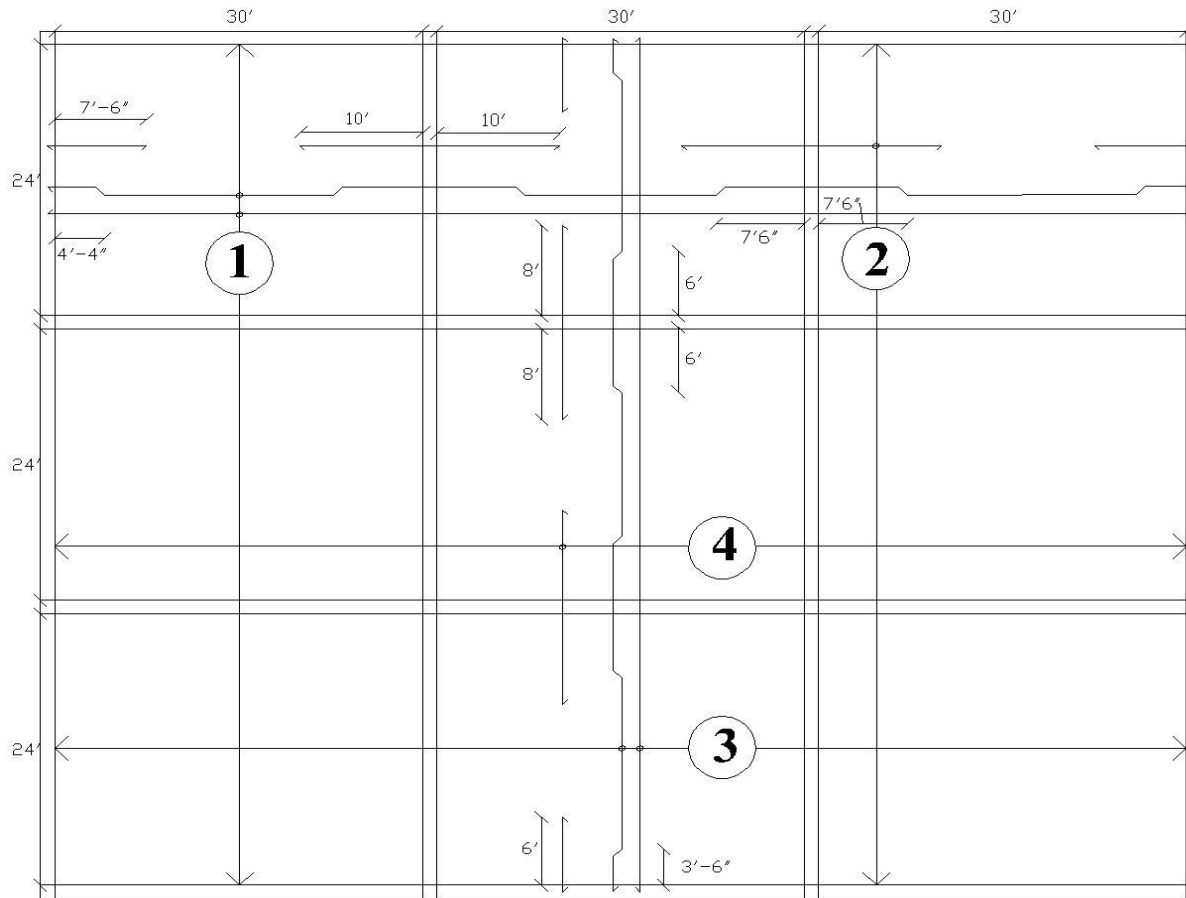
Using $\Phi 10$ bar @ $5.27' \approx 5" \text{ c/c}$ at bottom along long direction crank 50% bar to negative zone.

$$-A_{S B} = \frac{11.716}{4.5 * (7.5 - 0.36)} = 0.365 \text{ in}^2/\text{ft}$$

$$\text{Already provided } A_{s1} = \frac{0.121 * 12}{10} = 0.145 \text{ in}^2/\text{ft}$$

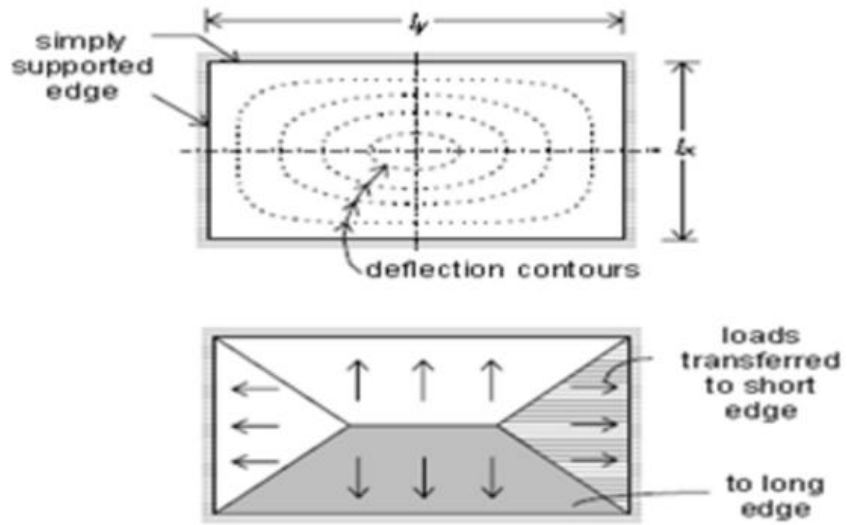
$$\text{Extra top required, } A_{s2} = (0.365 - 0.1452) \text{ in}^2/\text{ft} = 0.2198 \text{ in}^2/\text{ft}$$

Using $\Phi 10$ bar @ $6.6" \approx 6.5" \text{ c/c}$ extra top.



- Legends:**
- (1) $\text{Ø } 10 @ 5" \text{ c/c}$ althrough cranked alternatively.
 - (2) $\text{Ø } 10 @ 6.5" \text{ c/c}$ extra top.
 - (3) $\text{Ø } 10 @ 5" \text{ c/c}$ althrough cranked alternatively.
 - (4) $\text{Ø } 10 @ 5" \text{ c/c}$ extra top.
- All beams are 14"×14"
Slab Thickness= 8.5"

Figure: Reinforcement details of slab in plan.



(b) two-way slab action

