

Two Way Beam Supported Slab

References:

1. Design of Reinforced Concrete, 2014, 9th Edition, ACI 318-11 Code Edition, by Jack C. McCormac. Clemson University. Russell H. Brown. Clemson University
2. Design of Concrete Structures 14th Edition, 2009, by Arthur H. Nilson. Professor Emeritus. College of Engineering. Cornell University, David Darwin (University of Kansas), Charles Dolan (University of Wyoming)
3. Others

Types of slabs

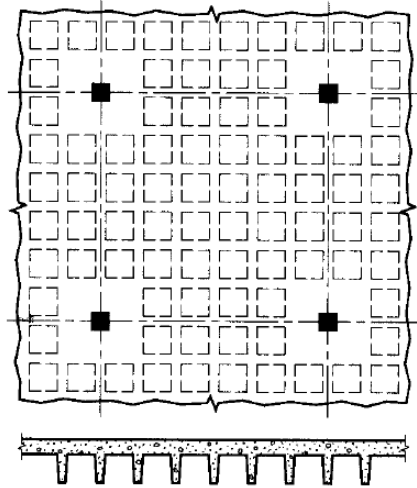
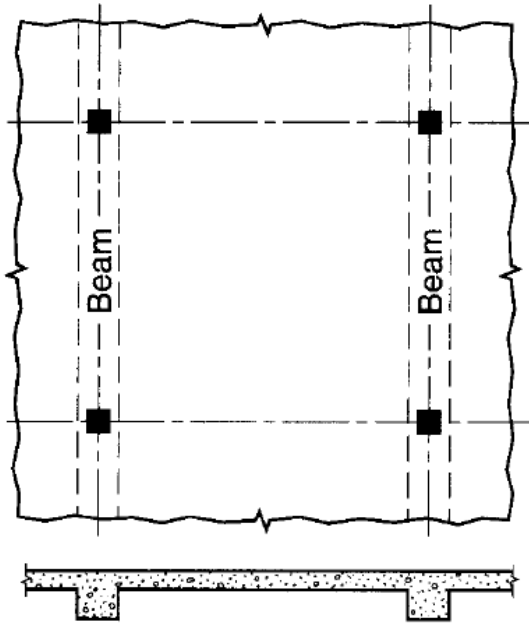
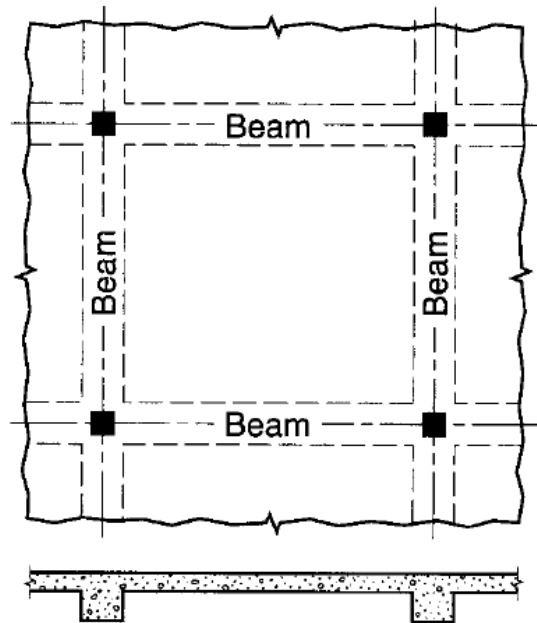


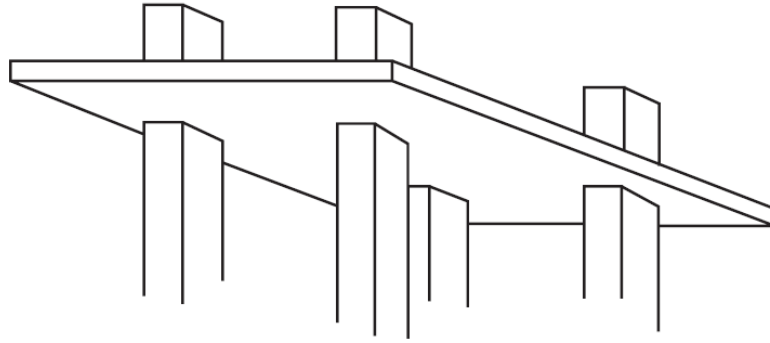
Figure: Grid or Waffle slab



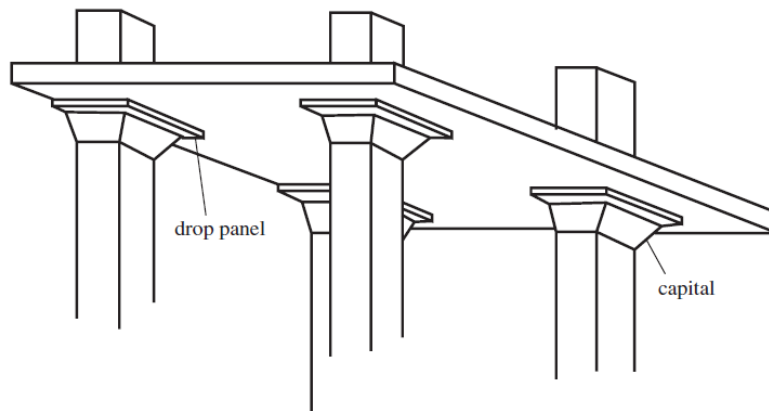
One way slab



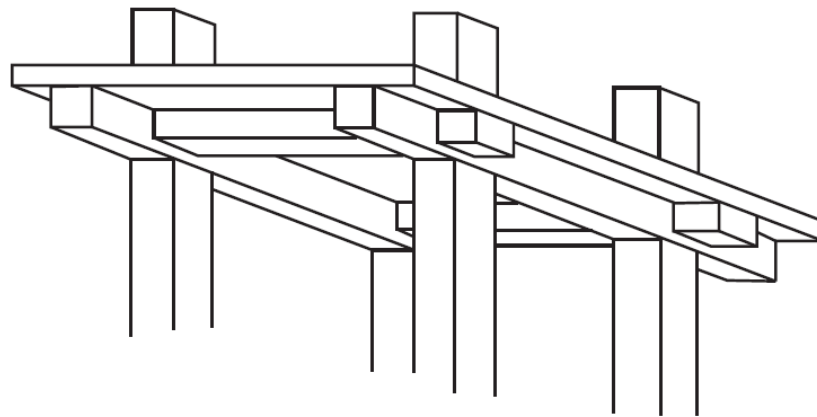
Two way slab



(b) Flat plate



(c) Flat slab



(d) Two-way slab with beams

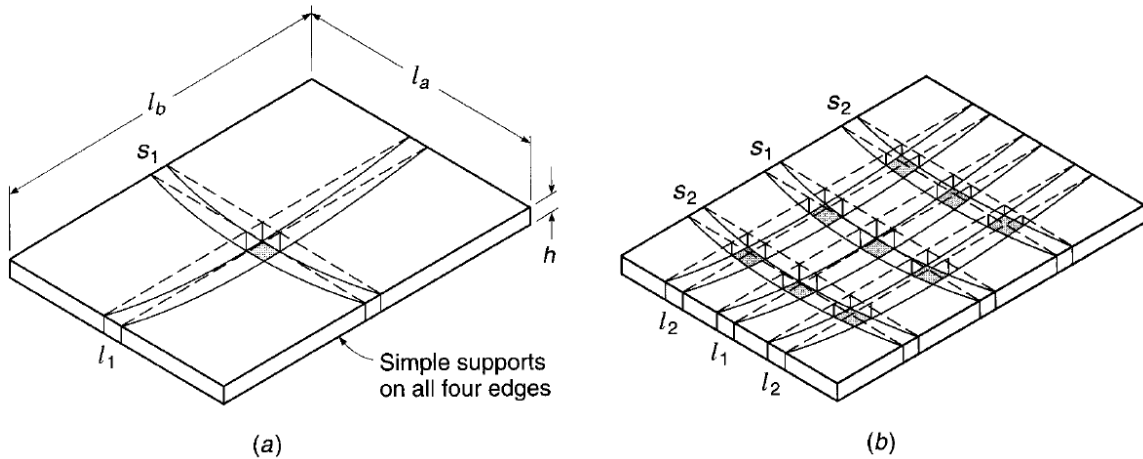


Figure: Two way slab (a) Bending of center strip, (b) grid model

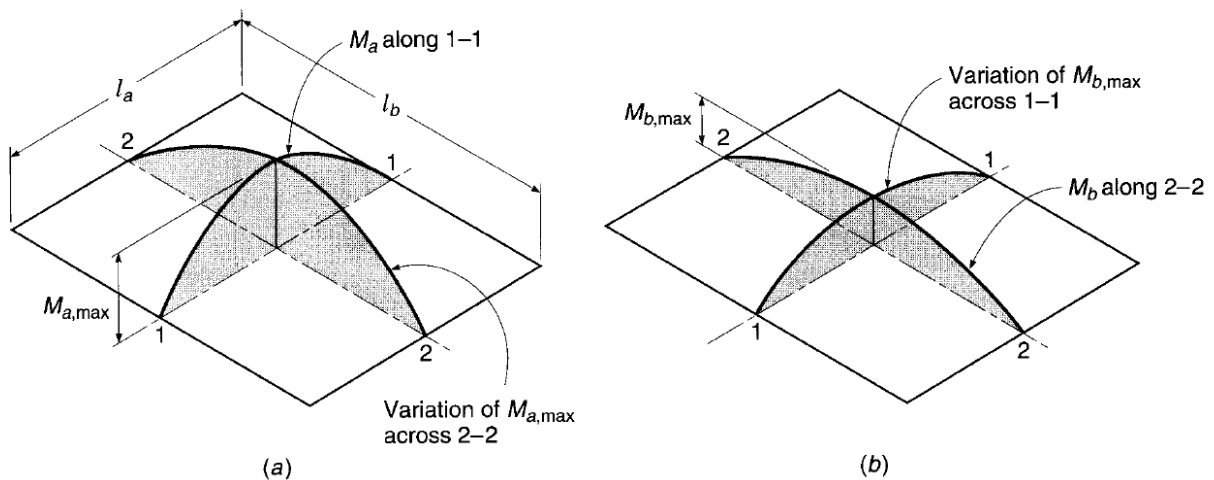


Figure: Moment variations of a uniformly loaded slab with simple supports on four sides.

Two way Slabs with Beams on All Sides:

The parameter used to define the relative stiffness of the beam and slab spanning in either direction is α , calculated from $\alpha = \frac{E_{cb} I_b}{E_{cs} I_s}$.

In which E_{cb} and E_{cs} are the modulus of elasticity of the beam and slab concrete (usually the same) and I_b and I_s are the moments of inertia of the effective beam and the slab.

Then α_m is defined as the average value of α for all beams on the edges of a given panel.

Minimum Thickness for two way slabs:

9.5.3.3 — For slabs with beams spanning between the supports on all sides, the minimum thickness, h , shall be as follows:

(a) For α_{fm} equal to or less than 0.2, the provisions of 9.5.3.2 shall apply;

(b) For α_{fm} greater than 0.2 but not greater than 2.0, h shall not be less than

$$h = \frac{\ell_n \left(0.8 + \frac{f_y}{200,000} \right)}{36 + 5\beta(\alpha_{fm} - 0.2)} \quad (9-12)$$

and not less than 5 in.;

(c) For α_{fm} greater than 2.0, h shall not be less than

$$h = \frac{\ell_n \left(0.8 + \frac{f_y}{200,000} \right)}{36 + 9\beta} \quad (9-13)$$

and not less than 3.5 in.;

(1) According to **ACI code 9.5.3.3**, for α_m equal to or less than 0.2, the minimum thickness of **ACI Table 9.5(c)**: shall apply.

ACI Table 9.5(c): Minimum thickness of slabs without interior beams

Yield Stress f_y , psi	Without Drop Panels			With Drop Panels		
	Exterior Panels		Interior Panels	Exterior Panels		Interior Panels
	Without Edge Beams	With Edge Beams ^a		Without Edge Beams	With Edge Beams ^a	
40,000	$l_n/33$	$l_n/36$	$l_n/36$	$l_n/36$	$l_n/40$	$l_n/40$
60,000	$l_n/30$	$l_n/33$	$l_n/33$	$l_n/33$	$l_n/36$	$l_n/36$
75,000	$l_n/28$	$l_n/31$	$l_n/31$	$l_n/31$	$l_n/34$	$l_n/34$

^a Slabs with beams along exterior edges. The value of α for the edge beam shall not be less than 0.8.

(2) For α_m greater than 0.2 but not greater than 2.0, the slab thickness must not be less than

$$h = \frac{\ell_n \left[0.8 + \left(\frac{f_y}{200,000} \right) \right]}{36 + 5\beta(\alpha_m - 0.2)} \text{ and not less than 5.0 inch..... (01)}$$

(3) For α_m greater than 2.0, the thickness must not be less than

$$h = \frac{l_n \left[0.8 + \left(\frac{f_y}{200,000} \right) \right]}{36 + 9\beta} \text{ and not less than 3.5 inch..... (02)}$$

where l_n = clear span in long direction, inches

α_m = average value of α for all beams on edges of a panel.

β = ratio of clear span in long direction to clear span in short direction.

f_y = Yield strength of steel in psi

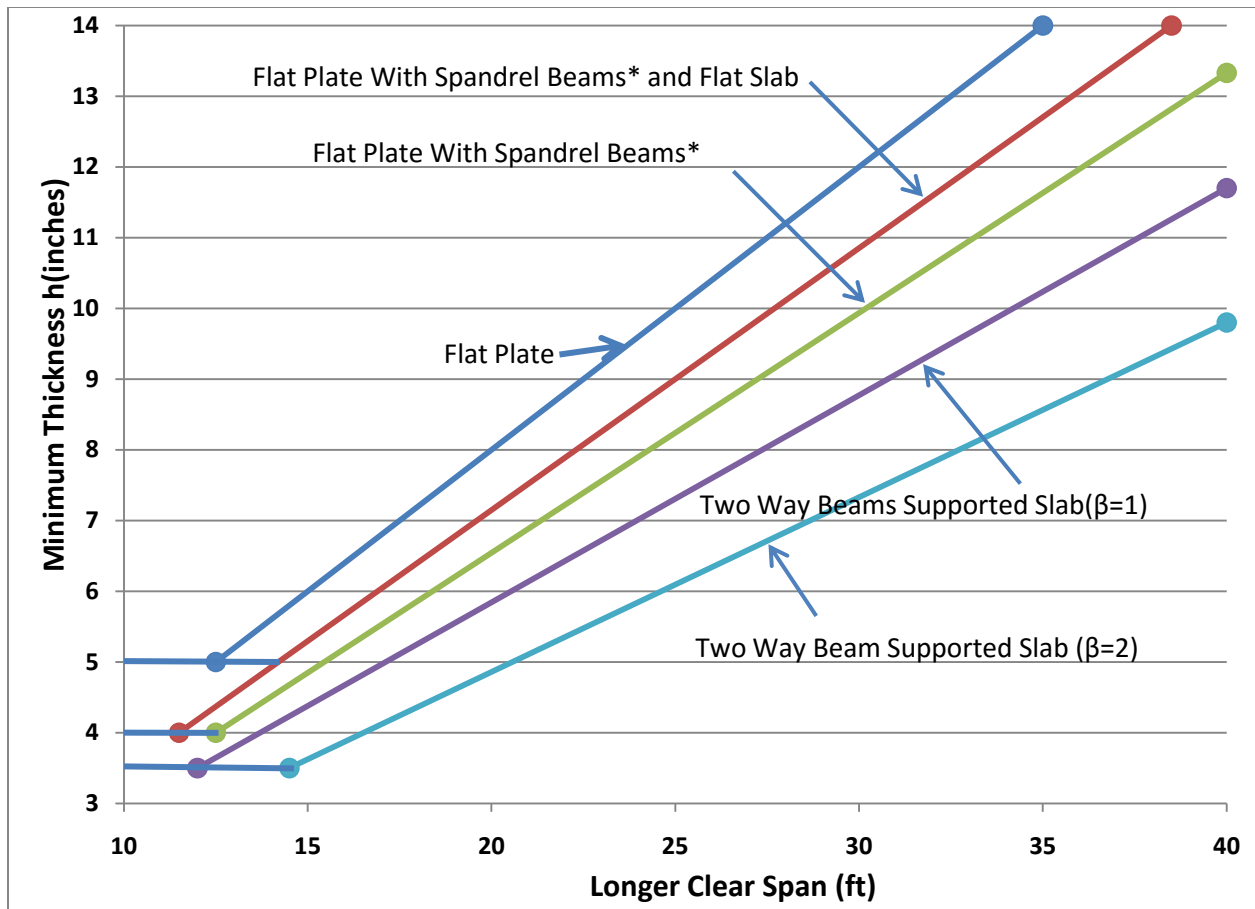
Note: At discontinuous edges, an edge beam must be provided with a stiffness ratio α not less than 0.8; otherwise the minimum thickness provided by Eq. (01) or (02) must be increased by at least 10 percent in the panel with the discontinuous edge.

Additional Notes:

In all cases, slab thickness less than stated minimum may be used if it can be shown by computation that deflections will not exceed the limit values of ACI Table 9.5 (b).

ACI Table 9.5 (b): Maximum allowable computed deflections

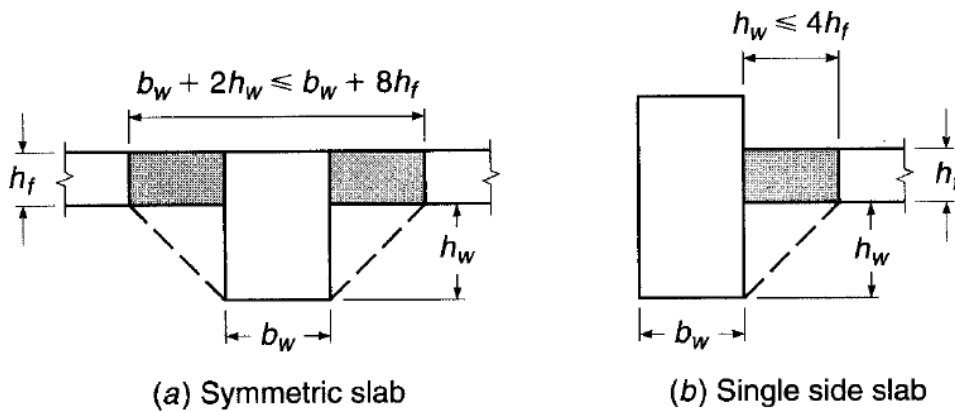
Type of member	Deflection to be considered	Deflection Limitation
Flat roofs not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to the live load (LL)	$\frac{l}{180}$
Floors not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to the live load (LL)	$\frac{l}{360}$
Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections	That part of the total deflection occurring after attachment of the nonstructural elements (sum of the long-time deflection due to all sustained loads and the immediate deflection due to any additional live load)	$\frac{l}{480}$
Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections		$\frac{l}{240}$



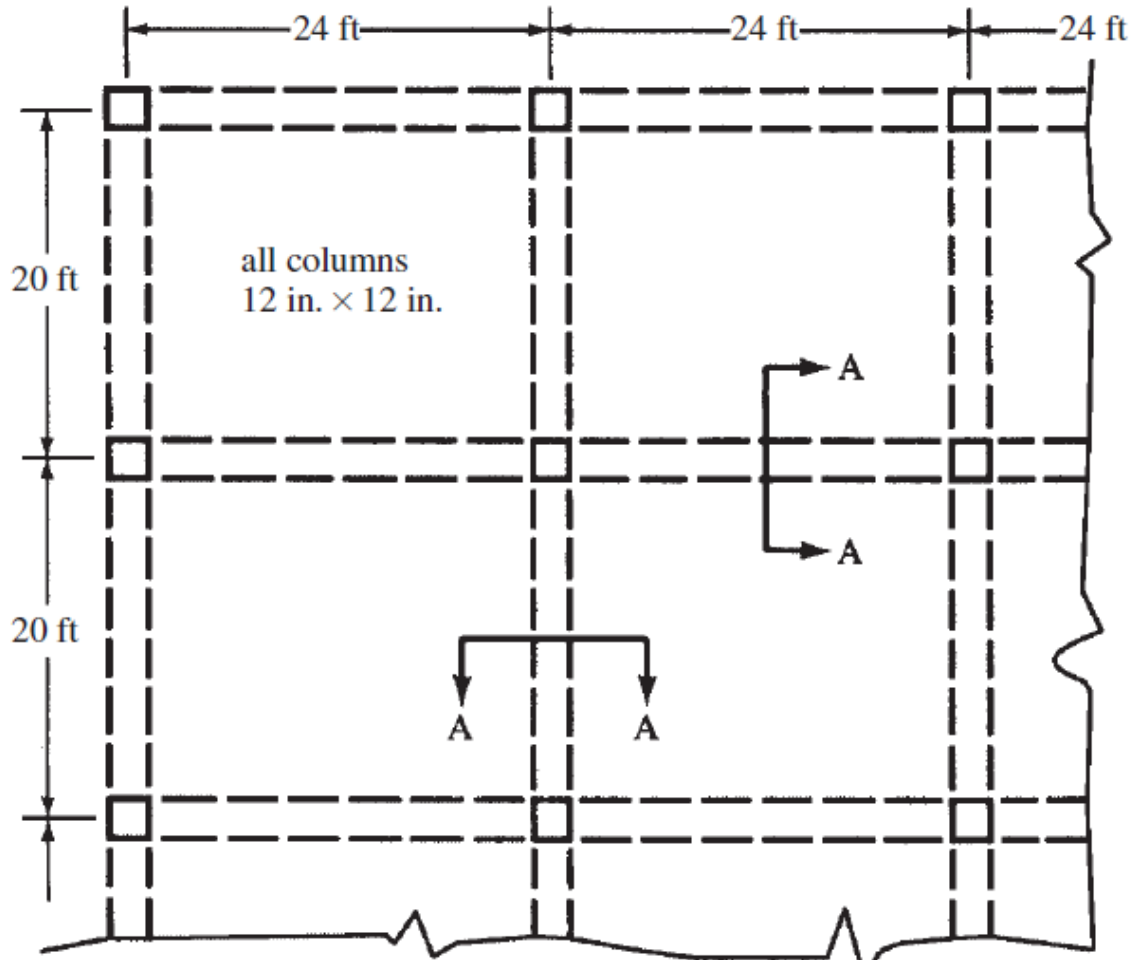
Finding the slab thickness for two way slab with edge beams

Approximate Alternate Method for determining the slab thickness for two way slab with edge beams

For grade 60 steel, thickness, h, inches = P / 160 [note: P=, Perimeter in inches]



Example: The two-way slab shown in Figure below has been assumed to have a thickness of 7 in. Section A–A in the figure shows the beam cross section. Check the ACI equations to determine if the slab thickness is satisfactory for an interior panel. $f'_c = 3000$ psi, $f_y = 60,000$ psi, and normal-weight concrete.



Solution:

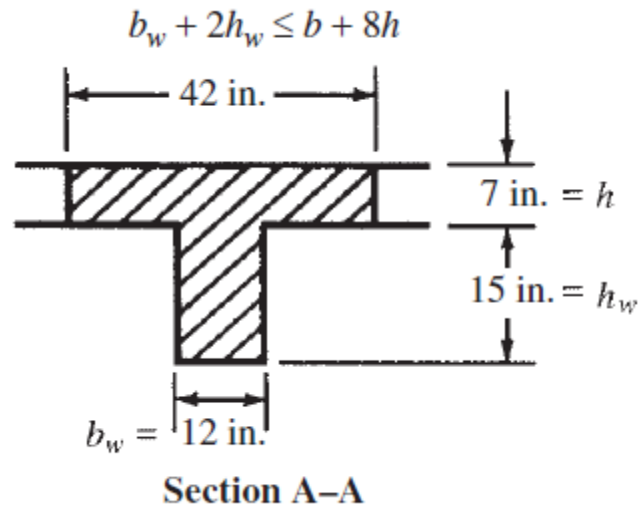
Computing α_1 for Long (Horizontal) Span for Interior Beams

I_s = gross moment of inertia of slab 20 ft wide

$$= \left(\frac{1}{12} \right) (12 \text{ in./ft} \times 20 \text{ in.}) (7 \text{ in.})^3 = 6860 \text{ in.}^4$$

I_b = gross I of T-beam cross section shown in Figure about centroidal axis = 18,060 in.⁴

$$\alpha_1 = \frac{EI_b}{EI_s} = \frac{(E)(18,060 \text{ in.}^4)}{(E)(6860 \text{ in.}^4)} = 2.63$$



Computing α_2 for Long Interior Beams

$$I_s \text{ for 24-ft-wide slab} = \left(\frac{1}{12}\right) (12 \text{ in./ft} \times 24 \text{ in.}) (7 \text{ in.})^3 = 8232 \text{ in.}^4$$

$$I_b = 18,060 \text{ in.}^4$$

$$\alpha_2 = \frac{(E)(18,060 \text{ in.}^4)}{(E)(8232 \text{ in.}^4)} = 2.19$$

$$\alpha_{fm} = \frac{\alpha_1 + \alpha_2}{2} = \frac{2.63 + 2.19}{2} = 2.41$$

Determining Slab Thickness per ACI Section 9.5.3.3

$$\alpha_{fm} = 2.41 > 2$$

\therefore Use ACI Equation 9-13

$$h = \frac{\ell_n \left(0.8 + \frac{f_y}{200,000 \text{ psi}}\right)}{36 + 9\beta}$$

$$\ell_{n \text{ long}} = 24 \text{ ft} - 1 \text{ ft} = 23 \text{ ft}$$

$$\ell_{n \text{ short}} = 20 \text{ ft} - 1 \text{ ft} = 19 \text{ ft}$$

$$\beta = \frac{23 \text{ ft}}{19 \text{ ft}} = 1.21$$

$$h = \frac{(23 \text{ ft}) \left(0.8 + \frac{60,000 \text{ psi}}{200,000 \text{ psi}}\right)}{36 + (9)(1.21)} = 0.540 \text{ ft} = 6.47 \text{ in.}$$

Use 7-in. slab

Note that the interior panel will generally not control the required slab thickness. Usually it will be an edge or corner panel. The interior panel was chosen here to illustrate the calculations and to avoid excess complexity. Had a corner panel been selected, each edge of the panel would have had a different α_f .

The following example was done by Mr. Naim Hassan, 3rd Year 2nd Semester Student of CE Dept., AUST

Given a slab of 18 feet by 20 feet and supported on all four edges by beams width of 12 inches on two sides & 14 inches on other two sides, beams' depth 11 inches below the slab. The slab is a typical interior slab. $f_y=60$ ksi, $f'_c=3$ ksi; For this slab assume slab thickness 7 inches, if it is sufficient or not?

Solution:

$$h_f = \frac{(18+20) \times 12 \times 2}{145} = 6.5 \text{ inch}$$

$$\text{now, } \alpha_f = \frac{E_{cb} I_b}{E_{cs} I_s}$$

$$I_{s1,3} = \frac{18 \times 12 \times 6.5^3}{12} = 4943.25 \text{ in}^4$$

$$I_{s2,4} = \frac{20 \times 12 \times 6.5^3}{12} = 5492.50 \text{ in}^4$$

$$b_w + 2h_w \leq b_w + 8h_f \rightarrow 12 + 2 \times 11 \leq 12 + 8 \times 6.5$$

$$b_w + 2h_w = 34 \text{ in}$$

for beam the centroid is,

$$y_{1,3} = \frac{34 \times 6.5 \times \frac{6.5}{2} + 11 \times 12 \times (6.5 + \frac{11}{2})}{34 \times 6.5 + 11 \times 12} = 6.52 \text{ in}$$

$$I_{b1,3} = \frac{12 \times 11^3}{12} + 11 \times 12 \times ((6.5 + 11/2) - 6.52)^2 + \frac{34 \times 6.5^3}{12} + 34 \times 6.5 \times (6.5/2 - 6.52)^2 = 8436.25 \text{ in}^4$$

$$b_w + 2h_w = 14 + 2 \times 11 = 36 \text{ in}$$

$$y_{2,4} = \frac{36 \times 6.5 \times \frac{6.5}{2} + 11 \times 14 \times (6.5 + \frac{11}{2})}{36 \times 6.5 + 11 \times 14} = 6.72 \text{ in}$$

$$I_{b2,4} = \frac{14 \times 11^3}{12} + 11 \times 14 \times ((6.5 + 11/2) - 6.72)^2 + \frac{36 \times 6.5^3}{12} + 36 \times 6.5 \times (6.5/2 - 6.72)^2 = 9487.55 \text{ in}^4$$

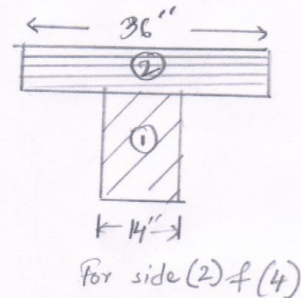
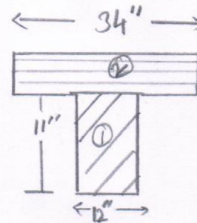
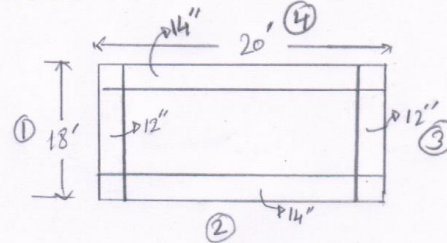
$$\alpha_{m1,3} = \frac{I_{b1,3}}{I_{s1,3}} = \frac{8436.25}{4943.25} = 1.71$$

$$\alpha_{m1,3} = \frac{I_{s2,4}}{I_{s2,4}} = \frac{9487.55}{5492.50} = 1.73$$

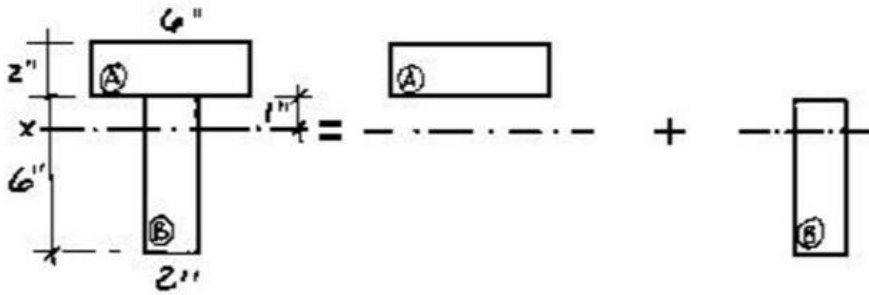
$$\alpha_{\text{mavg}} = \frac{1.71 + 1.71 + 1.73 + 1.73}{4} = 1.72$$

$$\text{Since } 0.2 < \alpha_{\text{mavg}} < 2 \quad ; \quad h_{\text{min}} = \frac{l_n (0.8 + \frac{f_y}{200000})}{36 + 5\beta(\alpha_{fm} - 0.2)} = \frac{18 \times 12 (0.8 + \frac{60000}{200000})}{36 + 5 \times \frac{18}{16} (1.72 - 0.2)} = 5.5 \text{ inch}$$

Hence, it is sufficient to provide 7 inches slab thickness.



Calculation of Inertia for T beam-Example:



$$I_{xx} = \text{sum} (I_c + Ad^2)$$

In this case:

$$I_{xx} = I_A + A_A d^2 + I_B + A_B d^2$$

$$I_{xx} = 1/12 (6\text{in}) (2\text{in})^3 + (12\text{in}^2) (2\text{in})^2 + 1/12 (2\text{in}) (6\text{in})^3 + 12\text{in}^2 (2\text{in})^2$$

$$I_{xx} = 4 \text{ in}^4 + 48 \text{ in}^4 + 36 \text{ in}^4 + 48 \text{ in}^4$$

$$I_{xx} = 134 \text{ in}^4$$

Alternate Design:

Design Procedure of Two-way Slabs using ACI Moment Coefficients:

Step 01: Determination of thickness of the slab panel.

Determine the thickness of the slab panel using previous article.

Step 02: Calculation of factored load.

$$W_u = 1.2 * DL + 1.6 * LL$$

where DL= Total dead load (i.e.: Slab self weight, Floor finish, Partition wall, Plaster etc.)

LL= Live load.

Step 03: Determination of moment coefficients.

$$m = \frac{A}{B}$$

where A= Shorter length of the slab.

B= Longer length of the slab.

Case type is identified from end condition. Using the value of 'm' corresponding moment coefficients are obtained for respective 'case type' from corresponding tables. The co-efficients are:










- $C_{A \text{ neg}}$ and $C_{B \text{ neg}}$
- $C_{A \text{ DL pos}}$ and $C_{B \text{ DL pos}}$
- $C_{A \text{ LL pos}}$ and $C_{B \text{ LL pos}}$

Table for $C_{A \text{ neg}}$ and $C_{B \text{ neg}}$

Ratio $m = \frac{A}{B}$	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00		0.045		0.050	0.075	0.071		0.033	0.061
		0.045	0.076	0.050			0.071	0.061	0.033
0.95		0.050		0.055	0.079	0.075		0.038	0.065
		0.041	0.072	0.045			0.067	0.056	0.029
0.90		0.055		0.060	0.080	0.079		0.043	0.068
		0.037	0.070	0.040			0.062	0.052	0.025
0.85		0.060		0.066	0.082	0.083		0.049	0.072
		0.031	0.065	0.034			0.057	0.046	0.021
0.80		0.065		0.071	0.083	0.086		0.055	0.075
		0.027	0.061	0.029			0.051	0.041	0.017
0.75		0.069		0.076	0.085	0.088		0.061	0.078
		0.022	0.056	0.024			0.044	0.036	0.014
0.70		0.074		0.081	0.086	0.091		0.068	0.081
		0.017	0.050	0.019			0.038	0.029	0.011
0.65		0.077		0.085	0.087	0.093		0.074	0.083
		0.014	0.043	0.015			0.031	0.024	0.008
0.60		0.081		0.089	0.088	0.095		0.080	0.085
		0.010	0.035	0.011			0.024	0.018	0.006
0.55		0.084		0.092	0.089	0.096		0.085	0.086
		0.007	0.028	0.008			0.019	0.014	0.005
0.50		0.086		0.094	0.090	0.097		0.089	0.088
		0.006	0.022	0.006			0.014	0.010	0.003










*A cross-hatched edge indicates that the slab continues across or is fixed at the support; an unmarked edge indicates a support at which torsional resistance is negligible.

Table 04: Table for $C_{A DL pos}$ and $C_{B DL pos}$

Ratio $m = \frac{A}{B}$	Case 1 	Case 2 	Case 3 	Case 4 	Case 5 	Case 6 	Case 7 	Case 8 	Case 9 	
1.00	$C_{A DL}$	0.036	0.018	0.018	0.027	0.027	0.033	0.027	0.020	0.023
	$C_{B DL}$	0.036	0.018	0.027	0.027	0.018	0.027	0.033	0.023	0.020
0.95	$C_{A DL}$	0.040	0.020	0.021	0.030	0.028	0.036	0.031	0.022	0.024
	$C_{B DL}$	0.033	0.016	0.025	0.024	0.015	0.024	0.031	0.021	0.017
0.90	$C_{A DL}$	0.045	0.022	0.025	0.033	0.029	0.039	0.035	0.025	0.026
	$C_{B DL}$	0.029	0.014	0.024	0.022	0.013	0.021	0.028	0.019	0.015
0.85	$C_{A DL}$	0.050	0.024	0.029	0.036	0.031	0.042	0.040	0.029	0.028
	$C_{B DL}$	0.026	0.012	0.022	0.019	0.011	0.017	0.025	0.017	0.013
0.80	$C_{A DL}$	0.056	0.026	0.034	0.039	0.032	0.045	0.045	0.032	0.029
	$C_{B DL}$	0.023	0.011	0.020	0.016	0.009	0.015	0.022	0.015	0.010
0.75	$C_{A DL}$	0.061	0.028	0.040	0.043	0.033	0.048	0.051	0.036	0.031
	$C_{B DL}$	0.019	0.009	0.018	0.013	0.007	0.012	0.020	0.013	0.007
0.70	$C_{A DL}$	0.068	0.030	0.046	0.046	0.035	0.051	0.058	0.040	0.033
	$C_{B DL}$	0.016	0.007	0.016	0.011	0.005	0.009	0.017	0.011	0.006
0.65	$C_{A DL}$	0.074	0.032	0.054	0.050	0.036	0.054	0.065	0.044	0.034
	$C_{B DL}$	0.013	0.006	0.014	0.009	0.004	0.007	0.014	0.009	0.005
0.60	$C_{A DL}$	0.081	0.034	0.062	0.053	0.037	0.056	0.073	0.048	0.036
	$C_{B DL}$	0.010	0.004	0.011	0.007	0.003	0.006	0.012	0.007	0.004
0.55	$C_{A DL}$	0.088	0.035	0.071	0.056	0.038	0.058	0.081	0.052	0.037
	$C_{B DL}$	0.008	0.003	0.009	0.005	0.002	0.004	0.009	0.005	0.003
0.50	$C_{A DL}$	0.095	0.037	0.080	0.059	0.039	0.061	0.089	0.056	0.038
	$C_{B DL}$	0.006	0.002	0.007	0.004	0.001	0.003	0.007	0.004	0.002

*A cross-hatched edge indicates that the slab continues across or is fixed at the support; an unmarked edge indicates a support at which torsional resistance is negligible.

Table: Table for $C_{A LL pos}$ and $C_{B LL pos}$

Ratio $m = \frac{A}{B}$		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
										
1.00	$C_{A LL}$	0.036	0.027	0.027	0.032	0.032	0.035	0.032	0.028	0.030
	$C_{B LL}$	0.036	0.027	0.032	0.032	0.027	0.032	0.035	0.030	0.028
0.95	$C_{A LL}$	0.040	0.030	0.031	0.035	0.034	0.038	0.036	0.031	0.032
	$C_{B LL}$	0.033	0.025	0.029	0.029	0.024	0.029	0.032	0.027	0.025
0.90	$C_{A LL}$	0.045	0.034	0.035	0.039	0.037	0.042	0.040	0.035	0.036
	$C_{B LL}$	0.029	0.022	0.027	0.026	0.021	0.025	0.029	0.024	0.022
0.85	$C_{A LL}$	0.050	0.037	0.040	0.043	0.041	0.046	0.045	0.040	0.039
	$C_{B LL}$	0.026	0.019	0.024	0.023	0.019	0.022	0.026	0.022	0.020
0.80	$C_{A LL}$	0.056	0.041	0.045	0.048	0.044	0.051	0.051	0.044	0.042
	$C_{B LL}$	0.023	0.017	0.022	0.020	0.016	0.019	0.023	0.019	0.017
0.75	$C_{A LL}$	0.061	0.045	0.051	0.052	0.047	0.055	0.056	0.049	0.046
	$C_{B LL}$	0.019	0.014	0.019	0.016	0.013	0.016	0.020	0.016	0.013
0.70	$C_{A LL}$	0.068	0.049	0.057	0.057	0.051	0.060	0.063	0.054	0.050
	$C_{B LL}$	0.016	0.012	0.016	0.014	0.011	0.013	0.017	0.014	0.011
0.65	$C_{A LL}$	0.074	0.053	0.064	0.062	0.055	0.064	0.070	0.059	0.054
	$C_{B LL}$	0.013	0.010	0.014	0.011	0.009	0.010	0.014	0.011	0.009
0.60	$C_{A LL}$	0.081	0.058	0.071	0.067	0.059	0.068	0.077	0.065	0.059
	$C_{B LL}$	0.010	0.007	0.011	0.009	0.007	0.008	0.011	0.009	0.007
0.55	$C_{A LL}$	0.088	0.062	0.080	0.072	0.063	0.073	0.085	0.070	0.063
	$C_{B LL}$	0.008	0.006	0.009	0.007	0.005	0.006	0.009	0.007	0.006
0.50	$C_{A LL}$	0.095	0.066	0.088	0.077	0.067	0.078	0.092	0.076	0.067
	$C_{B LL}$	0.006	0.004	0.007	0.005	0.004	0.005	0.007	0.005	0.004

*A cross-hatched edge indicates that the slab continues across or is fixed at the support; an unmarked edge indicates a support at which torsional resistance is negligible.

Step 04: Calculation of moments.

Positive moments:

$$+M_A = C_{A DL} \times W_{DL} \times L_A^2 + C_{A LL} \times W_{LL} \times L_A^2;$$

$$+M_B = C_{B DL} \times W_{DL} \times L_B^2 + C_{B LL} \times W_{LL} \times L_B^2.$$

Negative Moments:

$$-M_A = C_{A, neg} \times W_T \times L_A^2;$$

$$-M_B = C_{B, neg} \times W_T \times L_B^2;$$

$$W_{DL} \times L_A^2 + C_{A LL} \times W_{LL}$$

W_{LL} = Uniform Live load per unit area, W_{DL} = Uniform Dead load per unit area

W_T = Total Uniform load per unit area = $W_{LL} + W_{DL}$

Start with Max moment, M then,

$$A_s = \frac{M}{.9 * f_y * (d - \frac{a}{2})}$$

Now, find $a = \frac{A_s * f_y}{0.85 * f'c * b}$

Then, do at least another trial, with new a, and find area of steel.

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