Slab Analysis Methods:

1. Elastic analysis- a concrete slab may be treated as an elastic plate. The flexure, shear, and deflection may be calculated by the fourth differential equation relating load to deflection for thin plates with small displacements, as presented by Timoshenko and Krieger. Also finite element can be used. In the finite element method, the slab is divided into a mesh of triangles or quadrilaterals. The displacement functions of the nodes (intersecting mesh points) are usually established, and the stiffness matrices are developed for computer analysis.
2. Elastic-plastic analysis or Non-linear Analysis
   – Finite element analysis (FEA) - when the finite element method takes into consideration the nonlinearity of the stress–strain relationship of the individual elements.
3. Approximate methods of analysis
   a. Direct design method (ACI)
   b. Equivalent frame method (ACI)
   c. Assignment of moments
4. Plastic analysis or Limit analysis: For plastic analysis, three methods are available. The yield line method was developed by Johansen to determine the limit state of the slab by considering the yield lines that occur in the slab as a collapse mechanism. The strip method was developed by Hillerborg. The slab is divided into strips, and the load on the slab is distributed in two orthogonal directions. The strips are analyzed as simple beams. The third method is optimal analysis. Dhir, Munday and others presented methods for minimizing reinforcement based on plastic analysis. Optimal solutions are complex in analysis and produce complex patterns of reinforcement.

The **Equivalent Frame Method**, EFM (ACI Code, Section 13.7), is one in which a three dimensional building is divided into a series of two-dimensional equivalent frames by cutting the building along lines midway between columns. The resulting frames are considered separately in the longitudinal and transverse directions of the building and treated floor by floor.

Equivalent frame method may be used in those cases where:

• slab layout is irregular and those not comply with the restrictions stated in Direct Design Method
• Where horizontal loading is applied to the structure
• Where partial loading patterns are significant because of the nature of the loading
• High live load/dead load ratios.

Direct Design Method: The direct design method, DDM (ACI Code, Section 13.6), is an approximate procedure for the analysis and design of two-way slabs. It is limited to slab systems subjected to uniformly distributed loads and supported on equally or nearly equally spaced columns. The method uses a set of coefficients to determine the design moments at critical sections.

In the Direct Design Method, the Static moment, M_0, is calculated for each panel. This moment is then divided between positive and negative moment regions using arbitrary moment coefficients and the positive moments are adjusted to reflect pattern loadings. In the Elastic Frame Method, all of this is accomplished by frame analyses.

Limitations of Direct Design Method -DDM:

Introduction

ACI318 Code provides two design procedures for slab systems:

13.6.1 Direct Design Method (DDM) For slab systems with or without beams loaded only by gravity loads and having a fairly regular layout meeting the following conditions:

13.6.1.1 There must be three or more spans in each directions.

13.6.1.2 Panels should be rectangular and the long span be no more than twice the short span.

13.6.1.3 Successive span lengths center-to-center of supports in each direction shall not differ by more than 1/3 of the longer span.

13.6.1.4 Columns must be near the corners of each panel with an offset from the general column line of no more 10% of the span in each direction.

13.6.1.5 The live load should not exceed 3 time the dead load in each direction. All loads shall be due gravity only and uniformly distributed over an entire panel.

13.6.1.6 If there are beams, there must be beams in both directions, and the relative stiffness of the beam in the two directions must be related as follows:

$$0.2 \le \frac{\alpha_1 l_2^2}{\alpha_2 l_1^2} \le 5.0$$

where

$$\alpha = \frac{E_{cb} I_b}{E_{cs} I_s}$$

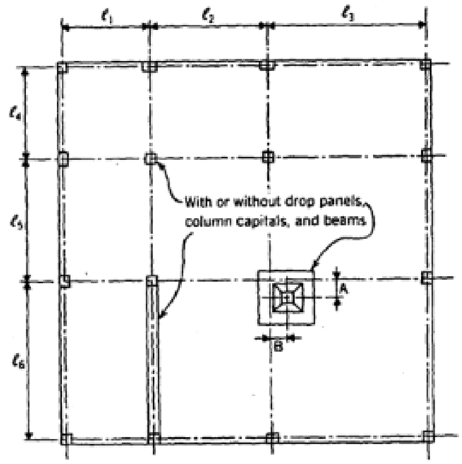
is the ratio of flexural stiffness of beam sections to flexural stiffness of a width of slab bounded laterally by center lines of adjacent panels (if any) on each side of the beam.

1. Three continuous spans in each direction, minimum.
2. Rectangular panels with aspect ratio ≤ 2.0. e.g., $\frac{\ell_6}{\ell_1} \le 2.0$.
3. Span lengths differ by $\frac{1}{3}$ or less of longer span. e.g., $\frac{\ell_5}{\ell_6} \ge 0.67$.
4. Column offset a maximum at 10%. e.g., $A \le 0.1\,\ell_6$.

5A. Gravity loads only, uniformly distributed, not lateral loads.

5B. Live load ≤ 3 × dead load.

6. Beam stiffness: $0.2 \le \frac{\alpha_1 \ell_2^2}{\alpha_2 \ell_1^2} \le 5.0$.
7. No moment redistribution as permitted by ACI Code Section 8.4.



Yield Line Theory: The theory is based on the principle that:
Work done in yield lines rotating = work done in loads moving

The yield line theory is largely based upon the yield lines that develop in any reinforced concrete slab (rectangular, circular, square or any other geometrical shape in plan) before its final collapse. This stage reaches under loads approaching collapse load or ultimate load that the slab can carry. The collapse loads, movements and shears can be calculated from the crack pattern developed in slab, under idealized support conditions and only uniformly distributed loads.

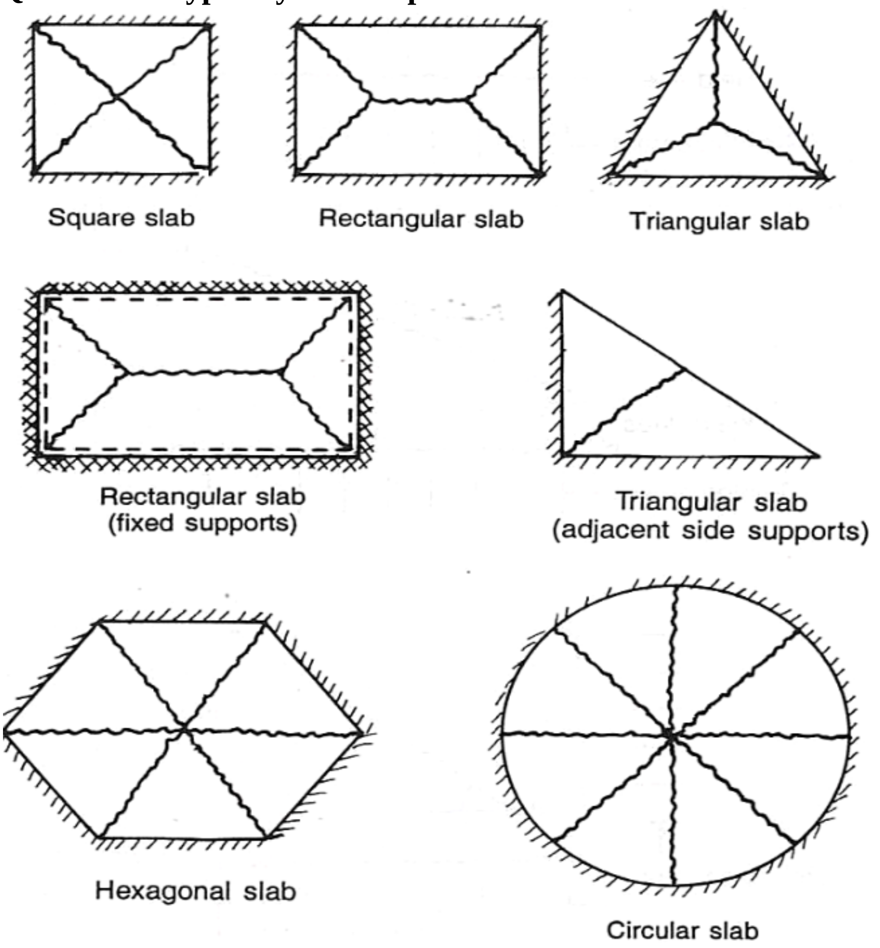
Yield Line Design demands familiarity with failure patterns, i.e. knowledge of how slabs might fail. This calls for a certain amount of experience, engineering judgement and confidence, none of which is easily gained. Yield Line Design tends to be a hand method. This may be seen as both an advantage and disadvantage. Each slab has to be judged on its merits and individually assessed. The method allows complex slabs to be looked at in a simple way, and, in an age of computers, it gives an independent method of analysis and verification.
Yield Line Analysis: Yield line theory permits prediction of the ultimate load of a slab system by postulating a collapse mechanism which is compatible with the boundary conditions. Slab sections are assumed to be ductile enough to allow plastic rotation to occur at critical section along yield lines.

1.Postulate a collapse mechanism compatible with the boundary conditions
2.Moment at plastic hinge lines ≈ Ultimate moment of resistance of the sections
3.Determine the ultimate load
4.Redistributions of bending moments are necessary with plastic rotations.

Q. What are the characteristic features of yield lines?

- *Yield lines end at the supporting edges of the slab*
- *Yield lines are straight*
- *A yield line or yield line produced passes through the intersection of the axes of rotation of adjacent slab*
- *Axes of rotation generally lie along lines of supports and pass over any columns.*

Q. Draw the typical yield line pattern for different slabs.



Typical yield line patterns in reinforced concrete slabs.