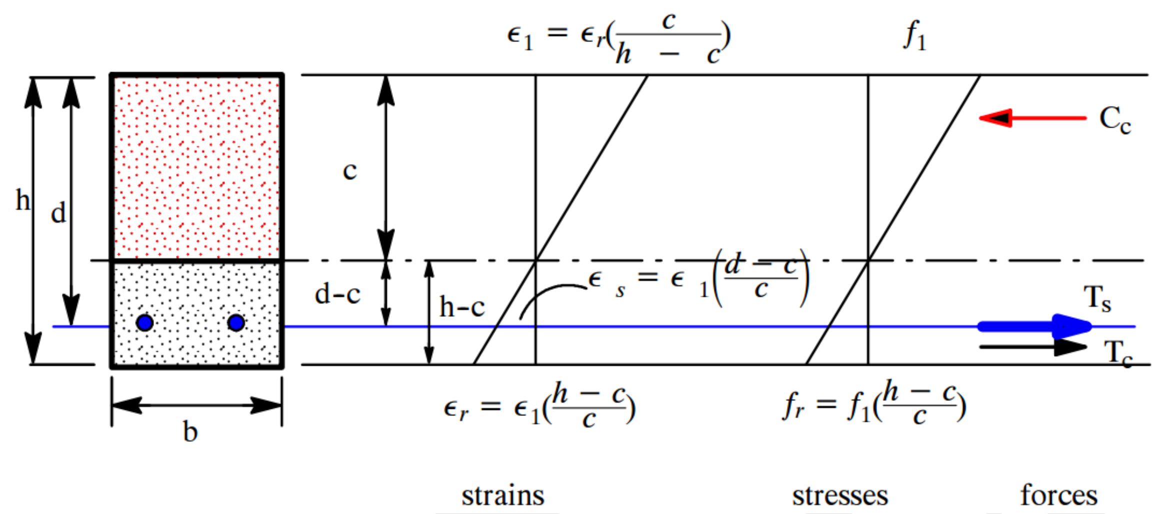
## Reinforced Concrete Beam - Uncracked

1. Assumptions:
    a. Strains vary linearly with distance from N.A.
    b. Linear stress-strain relationship;
    c. Strain compatibility between steel and concrete $\varepsilon = \varepsilon_s = \varepsilon_c$

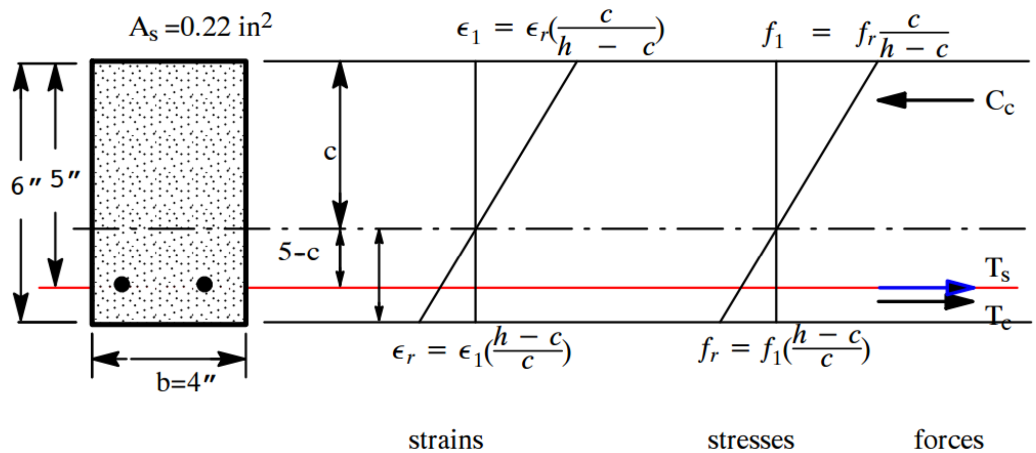


2. Reinforced concrete beam. Before cracking



**Example 1. Calculate Cracking Moment ($M_{cr}$)**

Calculate the moment of the section shown below when maximum tensile stress in concrete is equal to $f_r$ (Cracking Moment)




Given Material Properties

$f'_c$ = 3200 psi

$f_r$ = 500 psi = 0.5 ksi

$$E_c = 57,000\sqrt{3200} = 3,220,000 \ psi = 3,220 \ ksi$$

## Solution

$$\rho = \frac{A_s}{bd} = \frac{0.22(in^2)}{4(in) \times 5(in)} = 0.011$$

$$\frac{E_s}{E_c} = n = \frac{29,000 \ (ksi)}{3,220 \ (ksi)} = 9.01 \approx 9$$

$$\frac{h}{d} = \frac{6}{5} = 1.2$$

$$\frac{c}{d} = \frac{2\rho(n - 1) + (\frac{h}{d})^2}{2\rho(n - 1) + 2(\frac{h}{d})} = \frac{2 \times 0.011 \times 8 + (1.2)^2}{2 \times 0.011 \times 8 + 2 \times (1.2)} = 0.627$$

$$c = 0.627d = 0.627 \times 5(in) = 3.14 \ inches$$

$$f_1 = f_r\left(\frac{c}{h - c}\right) = 0.5 \times \frac{3.14}{6 - 3.14} = 0.549 \ ksi$$

Having $f_1$, we can easily calculate all forces:

$$C_c = \frac{1}{2}f_1cb = \frac{1}{2} \times 0.549(ksi) \times 3.14(in) \times 4(in) = 3.45 \ kip$$

$$T_s = f_1\left(\frac{d - c}{c}\right)(n - 1)A_s$$

$$= 0.549(ksi) \times \frac{5(in) - 3.14(in)}{3.14(in)} \times (9 - 1) \times 0.22(in^2) = 0.57 \ kips$$

$$T_c = \frac{1}{2}f_1\left(\frac{h - c}{c}\right)(h - c)b$$

$$= \frac{1}{2} \times 0.549(ksi) \times \frac{[6(in) - 3.14(in)]^2}{3.14(in)} \times 4(in) = 2.86 \quad kips$$

Check equilibrium, does it satisfy $C_c = T_s + T_c$ ?

$C_c = 0.57 + 2.86 = 3.43$ kips $= C_c$ kips ; the difference is due to rounding error associated with calculating "c"

Calculate moment about N.A. ( or any point on the cross section)

| Force Kips | Moment Arm inches | Moment in-kips |
|---|---|---|
| $C_c = 3.45 \ kips$ | $\frac{2}{3}c = \frac{2}{3} \times 3.14 = 2.09$ | = 7.21 ↻ |
| $T_s = 0.57$ | $d - c = 5 - 3.14 = 1.86$ | = 1.06 ↻ |
| $T_c = 2.86$ | $\frac{2}{3}(h - c) = \frac{2}{3}(6 - 3.14) = 1.91$ | = 5.46 ↻ |

Total M = 13.73 in-kips