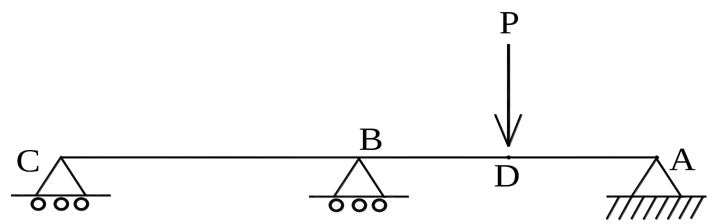
Indeterminate Beam Analysis by Virtual Work Method:

The indeterminate beam will be analyzed by virtual work method



Let,

Δ_c = Deflection at 'C' due to all causes

Δ_{co} = Deflection at 'C' due to actual load (S) while redundant ,extra support is removed

R_c = Reaction at C due to external loading

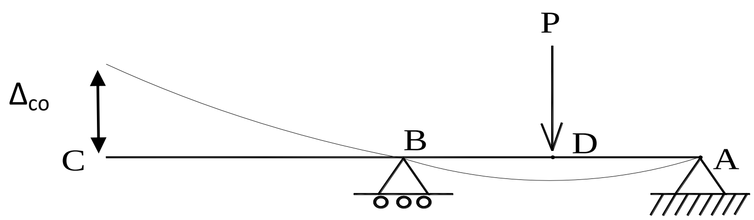
Δ_{cR} = Deflection at 'C' due to redundant reaction at C, R_c while actual load(s) is removed

δ_c = Deflection at 'C' due to unit load at C, therefore, $\Delta_{cR} = R_c . \delta_c$

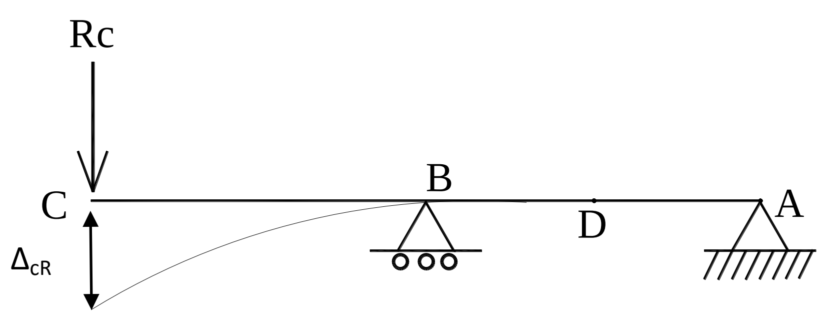
$\Delta_{cR} = R_c . \delta_c$

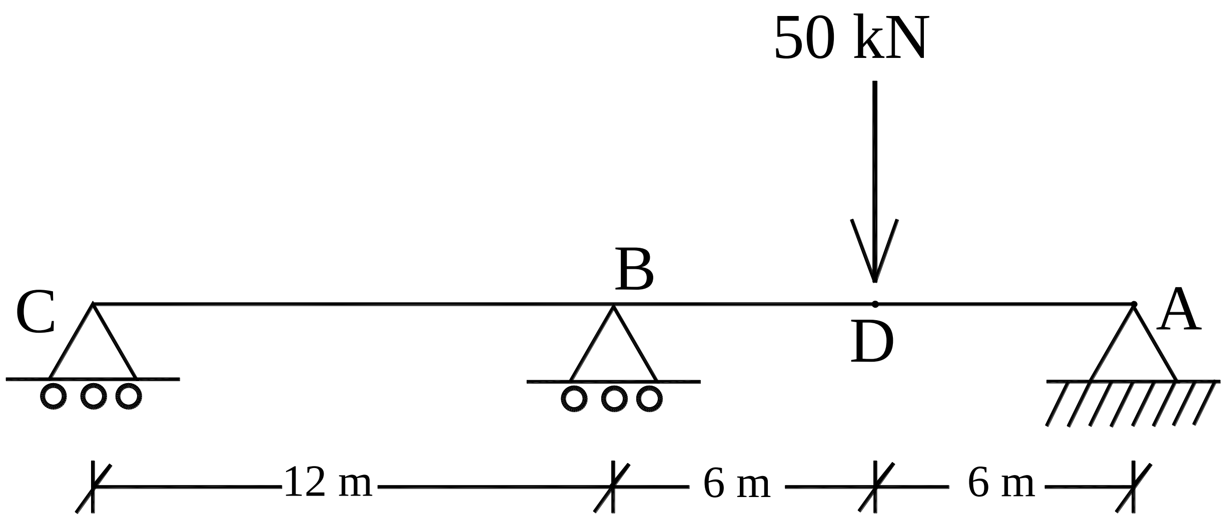
$\Delta_c = \Delta_{co} + \Delta_{cR} = \Delta_{co} + R_c \delta_c = 0$, where support deflection is zero, which is the ideal case.

Reaction at C, $R_c = - \Delta_{co} / \delta_c$



In the top figure, the redundant support is removed and in the following figure, the external load is removed and the elastic curves, i.e., the deflected beam-shapes are drawn for both instances.





Solution:

$R_c = - \Delta_{co} / \delta_c$

Where,

Δ_c = Deflection at 'C' due to all causes

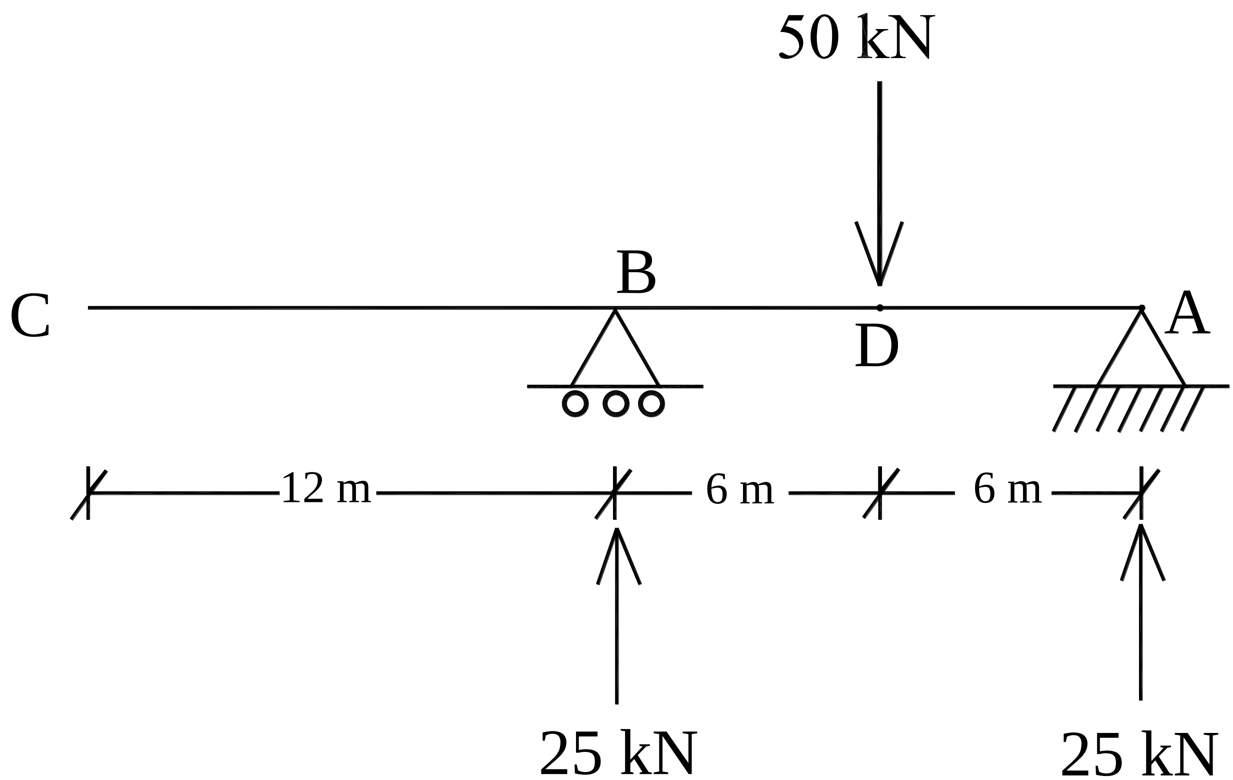
Δ_{co} = Deflection at 'C' due to actual load (S) while redundant ,extra support is removed

Δ_{cR} = Deflection at 'C' due to redundant reaction R_c while actual load (S) is removed

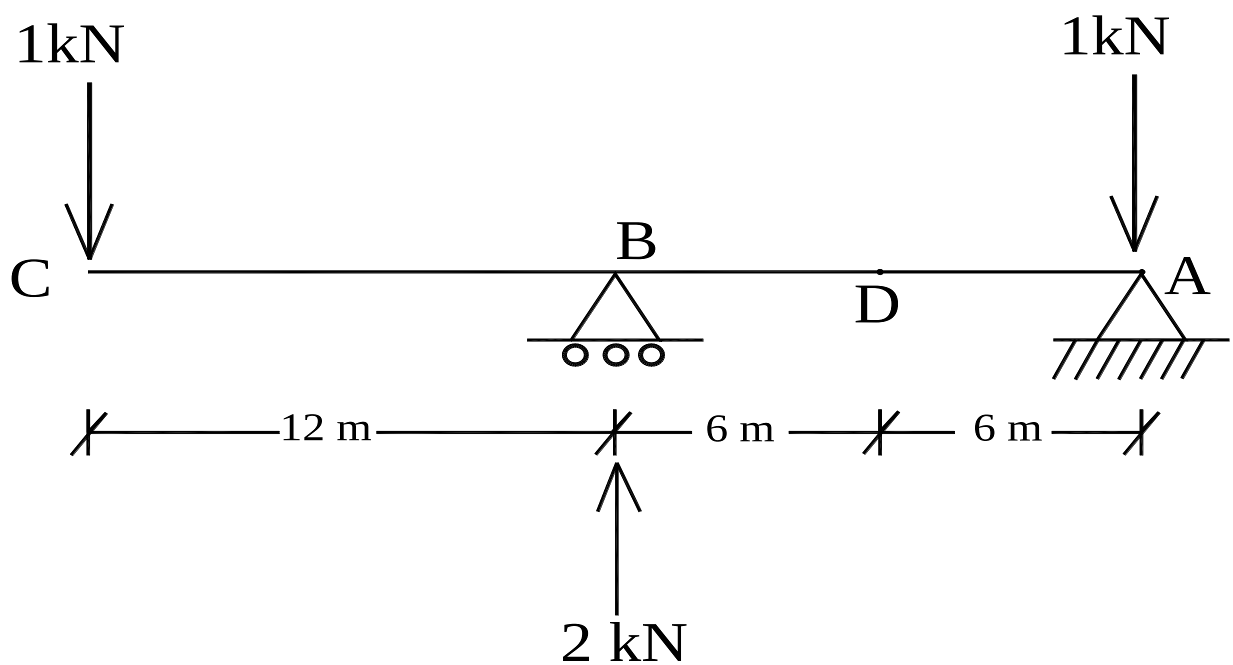
δ_c = Deflection at 'C' due to unit load at C

$\Delta_c = \Delta_{co} + R_c \, \delta_c$

$\Delta_c = \Delta_{co} + R_c \, \delta_c = 0, \quad R_c = \frac{-\Delta co}{\delta c}$



M_o diagram due to actual loading



M_1 diagram due to virtual loading

$EI.\Delta_c = \int_0^{6\ A\to D}\{25.x\}.\{-1.x\}dx + \int_0^{6\ D\to B}\{25.(x+6) - 50.x\}.\{-1.(x+6)\}dx$

$= -1800 + (-3600)$

$= -5400$

$\Delta_c = -\frac{5400}{EI}\ kN.m^3$

$\Delta_{co} = -\frac{5400}{EI}\ kN.m^3$

$EI.\ \delta_c = \int_0^{12\ A\to B}\frac{{M_1}^2}{EI}dx + \int_0^{12\ B\to C}\frac{{M_1}^2}{EI}dx$

$= \int_0^{12\ A\to B}(-1.x)^2\,dx + \int_0^{12\ B\to C}(-1.x)^2\,dx$

$= 576 + 576$

$= 1152$

$\delta_c = -\frac{1152}{EI}\ kN.m^3$

$R_c = -\frac{-\frac{5400}{EI}}{\frac{1152}{EI}}$

$= 4.69\ kN$

$\Sigma M_B = 0$, Let, clockwise +ve;

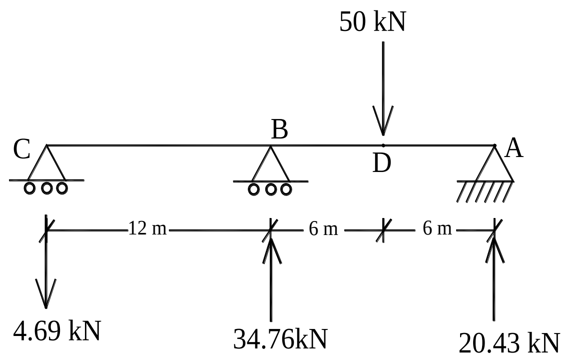
$50*6 - R_A*12 - 4.69*12 = 0$

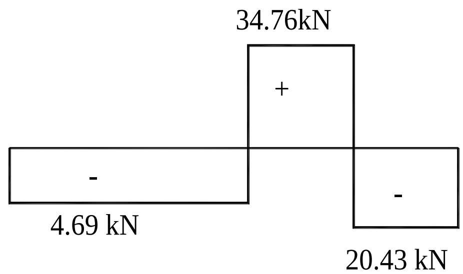
$R_A = 20.31$kN

$\Sigma F_x = 0$, ↑(+ve)

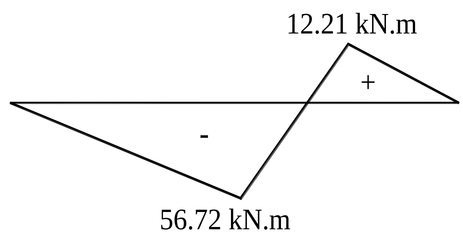
$20.31 - 4.69 - 50 + R_B = 0$

$R_B = 34.38$ kN (downward)





SFD



BMD

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