## Foundation or Footing Theories and Design: Part 2

Isolated Foundation or Footing transmit column load to the underlying soil.


Figure: Tributary area for columns shown in shades/colors
Design of Square Isolated Footings


Figure: Shapes of isolated footings.

Shear


Moments

The bending moment in a square reinforced concrete footing with a square column is the same about both axes because of symmetry. If the column is not square, the moment will be larger in the direction of the shorter column dimension. It should be noted, however, that the effective depth of the footing cannot be the same in the two directions because the bars in one direction rest on top of the bars in the other direction. The effective depth used for calculations might be the average for the two directions or, more conservatively, the value for the bars on top.
The critical section for bending is taken at the face of a reinforced concrete column or halfway between the middle and edge of a masonry wall or at a distance halfway from the edge of the base plate and the face of the column if structural steel columns are used.

## Two way or Punching Shear

As discussed in Section 13.10, the ACI Code equations ( $13.11 a, b, c$ ) give the nominal punching-shear strength on this perimeter:

$$
\begin{equation*}
V_{c}=4 \lambda \sqrt{f_{c}^{\prime}} b_{o} d \tag{16.7a}
\end{equation*}
$$

One way or Beam Shear

$$
V_{c}=2 \lambda \sqrt{f_{c}^{\prime}} b d
$$

The required depth of footing $d$ is then calculated from the usual equation

$$
V_{u} \leq \phi V_{c}
$$

Example.1: Design footings for the interior column of a building. Assume, base of footing location 5' below ground level . Permissible soil pressure, $\mathrm{q}_{\mathrm{a}=} \mathbf{6}$ ksf .Gravity loads: $\mathrm{P}_{\mathrm{DL}}=$ 351 kips , $P_{\text {LL }}=56.4$ kips, $\quad$ Service Moment, $M_{\text {service }}=75.4 \mathrm{k}-\mathrm{ft}$, column size = 16" $\times 16 "$

1.Design Data :

Assume, Weight of soil plus concrete above footing base $=135 \mathrm{pcf}$ (When, soil is wet packet use, weight of soil $=130 \mathrm{pcf}$; otherwise 100 pcf )

Interior column = 16 " x 16 " $\mathrm{f}^{\prime} \mathrm{c}=4000 \mathrm{psi}$ (for both footing and column), $\mathrm{fy}=60 \mathrm{ksi}$

## 2.Load Combination :

a) Gravity loads: from column, $\mathrm{P}_{\mathrm{DL}}=351 \mathrm{kips}, \mathrm{P}_{\mathrm{LL}}=56.4 \mathrm{kips}$

Service Moment for column, $\mathrm{M}_{\text {service }}=75.4 \mathrm{k}-\mathrm{ft}$

## 3. Base Area of Footing Calculation:

Weight of surcharge, soil plus concrete depth for footing $=(135 \mathrm{pcf} \times 5 \mathrm{ft} / 1000)=0.675 \mathrm{ksf}$
Total surcharge $=0.675+0.00$, extra surcharge if any $=0.675 \mathrm{ksf}$
so, Net permissible soil pressure $=(\mathbf{6}-\mathbf{0 . 6 7 5})=5.325 \mathrm{ksf}$
Area of footing, $\mathrm{A}_{\mathrm{f}}=\left(\frac{351+56.4}{5.325}\right)=76.5$ sq. ft.
Try $9 \mathrm{ft} \times 9 \mathrm{ft}$ square footing. ( $\mathrm{A}_{\mathrm{f}}=81 \mathrm{sq}$. ft .)
Now, we know, Sectional Modulus, $\mathrm{S}=\mathrm{I} / \mathrm{c}$, where c is the distance from the neutral axis to the most extreme fibre, $\mathrm{I}=$ moment of inertia, or, $\mathrm{S}=\mathrm{I} / \mathrm{c}=\left(\mathrm{bh}^{3} / 12\right) /(\mathrm{h} / 2)$
so, $S=b^{2} / 6$

$$
\mathbf{S}=\left(\frac{9 \times 9^{2}}{6}\right)=121.5 \mathrm{ft}^{3}
$$

As, $\mathrm{A}_{\mathrm{f}}=81 \mathrm{ft}^{2}$
Now, Stress, $\mathrm{q}=\frac{P}{A f}+\frac{M}{S f}=\frac{407.4}{81}+\frac{75.4}{121.5}=5.02+0.62=5.64 \mathrm{ksf}>5.325 \mathrm{ksf}($ Not OK $)$
Try, $9.5 \mathbf{f t} \times \mathbf{9 . 5} \mathbf{f t}$ square footing $\left(\mathbf{A}_{\mathbf{f}}=\mathbf{9 0 . 2 5} \mathbf{f t}^{\mathbf{2}}\right), \mathbf{S}=\left(\frac{9.5 \times 9.5^{2}}{6}\right)=121.5 \mathrm{ft}^{3}=\mathbf{1 4 2 . 9} \mathrm{ft}^{3}$

$$
\begin{aligned}
\mathrm{q} & =\frac{351+56.4}{90.25}+\frac{75.4}{142.9} \\
& =5.04 \mathrm{ksf}<5.325 \mathrm{ksf}(\mathrm{Ok})
\end{aligned}
$$

## 4. Footing Thickness :

Now, $\rho$ for bending $=A s / b d$ [Note: Minimum steel ratio for shrinkage and temperature in slab for 60 grade steel, is $\rho=0.0018$. Now assume, $\rho=0.002$ in the Rn equation below, since slab thickness, $\mathrm{h}>\mathrm{d}$ ]

$$
\begin{align*}
\mathrm{R}_{\mathrm{n}} & =\rho \mathrm{fy}\left(1-0.5 \rho f y /\left(0.85 f^{\prime} c\right)\right) \\
& =0.002 \times 60,000\left(1-\frac{0.5 \times 0.002 \times 60000}{0.85 \times 4000}\right) \\
& =117.9 \mathrm{psi} \\
\mathrm{~d}^{2}{ }_{\text {required }} & =\frac{M u}{\varphi R n}=\frac{M u \times 1000}{0.9 \times 117.9}=9.43 \mathrm{Mu} \ldots \ldots . . . . \tag{1}
\end{align*}
$$

However, $\mathrm{wL} .(\mathrm{L} / 2) \quad \mathbf{M u}=\left(\frac{P u}{A f}\right)\left(\frac{c^{2}}{2}\right) \ldots . .$. (2) $\quad\left[\mathrm{as},\left(\mathrm{Pu} / \mathrm{A}_{\mathrm{f}}\right)=\mathrm{w}=\mathrm{q}_{\mathrm{u}}\right][$ Note; wL.(L/2)]
Now, from eqn. (1), $\mathrm{d}^{2}$ req $=9.43 \mathrm{Mu}$

$$
\begin{aligned}
&=9.43 \times\left(\frac{P u}{A f}\right)\left(\frac{c^{2}}{2}\right)(\text { from eqn. } 2) \\
&=\sqrt{9.43 * \frac{P u}{A f} * \frac{c^{2}}{2}} \\
& \text { so, } \mathbf{d}_{\text {required }}=2.2 \mathbf{c} \sqrt{\frac{P u}{A f}}
\end{aligned}
$$

Now, bearing pressure for strength design $=$ Factored load $/$ Area of footing $=P_{u} / A_{f}$ $\mathrm{Pu}=1.2 \mathrm{DL}(351)+1.6 \mathrm{LL}(56.4)=511.44$
Bearing pressure $=511.44 / 90.25=5.67 \mathrm{ksf}$, or $\underline{\mathbf{q}}_{\underline{\underline{u}}} \underline{\text { net }}=5.67 \mathrm{ksf}$
Footing projection , $\mathbf{c}=\frac{\left(9.5-\frac{16}{12}\right)}{2} f t=4.08^{\prime}$
$\mathrm{h}=2.2 \mathrm{c} \sqrt{\frac{P u}{A f}}+4$ (considering, more than 3 " required clear cover + bar dia.)
$\mathrm{h}=2.2 * 4.08 * \sqrt{\frac{511.44}{90.25}}+4$
$=25.4 \mathrm{in}$
Try, $\mathrm{h}=27 \mathrm{in}$.
so, $\mathrm{d}=27-4=23 \mathrm{in}$.

## One way shear check:

## Considering beam Shear or One way shear :

Beam shear, $\mathrm{Vb}=\mathrm{qu} \times$ Beam strip length $\times(\mathrm{c}-\mathrm{d})$

$$
\begin{aligned}
& =5.67 \times 9.5 \times 2.167 \\
& =116.72 \mathrm{kips}
\end{aligned}
$$

Allowable shear, $\mathrm{Va}=2 \varphi \sqrt{f^{\prime} c}$ b d

$$
\begin{aligned}
& =(2 \times .75 \times \sqrt{ } 4000 \times(9.5 \times 12) \times 23) / 1000 \\
& =248.71 \mathrm{Kips}
\end{aligned}
$$

Since, $\mathrm{Va}>\mathrm{Vb}$, therefore, OK .


## Considering punching Shear or Two way shear :

Punching shear, $\mathrm{Vp}=$ Factored load, or $\mathrm{q}_{\mathrm{u}} \times$ Footing area - Punched out area $\times \mathrm{q}_{\mathrm{u}}$ $=9.5 \times 9.5 \times 5.67-[(39 \times 39) / 144] \times 5.67=451.55 \mathrm{Kips}$
Allowable shear, $\mathrm{Va}=4 \varphi \sqrt{ } f^{\prime} c$ b d

$$
\begin{aligned}
& =(4 * .75 * \sqrt{ } 4000 *(39 * 4) * 23) / 1000 \\
& =680.77 \mathrm{Kips}
\end{aligned}
$$

Since, $\mathrm{Va}>\mathrm{Vp}$, therefore OK.

The Bending Moment:
$\left.\mathrm{Mu}=\left(5.67 * 9.5^{*}(4.08)\right)^{\wedge} 2\right) / 2=448.3 \mathrm{k}-\mathrm{ft}$
[Note: $\mathrm{Mu}=\left(\mathrm{q}_{\mathrm{u}} \times \mathrm{b} \times \mathrm{c}^{2}\right) / 2$; considering moment for the whole width of footing]
Reinforcements : $\mathrm{A}_{\mathrm{s}}=\frac{M u}{\varnothing f y\left(d-\frac{a}{2}\right)}$
$=448.3 * 12 /[0.9 * 60 *(23-8 / 2)] ; \quad$ [assumed, $\mathrm{a}=8$ inches]
$=5.24 \mathrm{in}^{2}$;
$\mathrm{a}=\frac{A s f y}{.85 f^{\prime} c b}$
$=(5.43 * 60) / *(.85 * 4 * 12)==7.98$ inches; very close to 8 inches assumed, therefore, OK ;
$\mathrm{A}_{\mathrm{s}}$ required= $5.24 \mathrm{in}^{2}$.
Now, Minimum reinforcement for flexure,
$\mathrm{A}_{\mathrm{s}}($ minimum $)=200 * \mathrm{~b}^{*} \mathrm{~d} / \mathrm{fy}$

$$
=(200 * 9.5 * 12 * 23) / 60000=8.74 \mathrm{in}^{2}
$$

$\mathrm{A}_{\mathrm{s}}$ (minimum) $=8.74 \mathrm{in}^{2}$; within 9.5 ft footing width, controls.
Since, it is square footing, use 9 \#9 bar (Area provided $9 \mathrm{in}^{2}$ ) in each direction.


Figure: Detail of reinforcement
The reinforcement ratio is calculated based on rectangular section design, where the minimum reinforcement ratio $\rho_{\min }$ is not to be less than 0.0018 .


