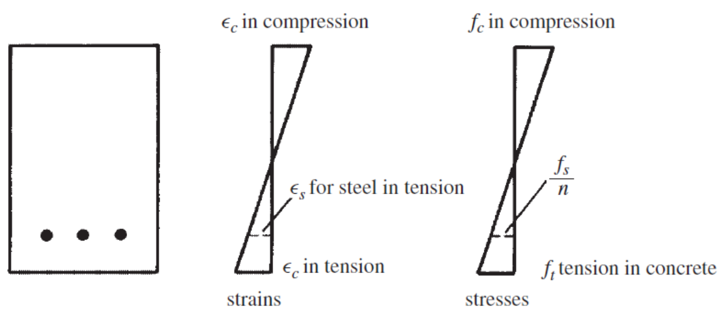
Flexure Analysis of a Beam and Cracking Moment

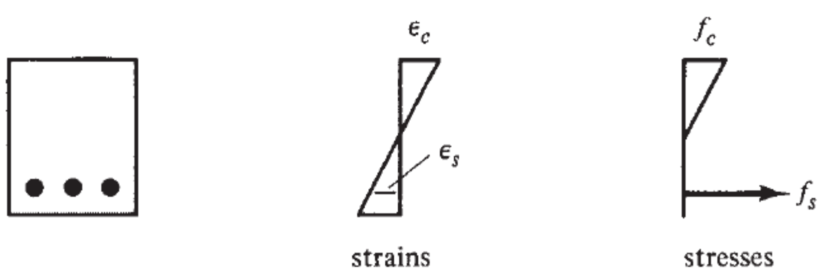
BEHAVIOR OF SIMPLY SUPPORTED REINFORCED CONCRETE BEAM LOADED TO FAILURE

Stage 1- Uncracked Concrete Stage: At minimal external load, Compressive stress at the top fibers was much less than the ultimate concrete compressive stress, f'_c . No cracks were observed at this stage. At small loads when the tensile stresses are less than the modulus of rupture (the bending tensile stress at which the concrete begins to crack), the entire cross section of the beam resists bending. Stress in the steel bars was equal to the stress in the adjacent concrete multiplied by the modular ratio, **n, the ratio of the modulus of elasticity of steel to that of concrete**.

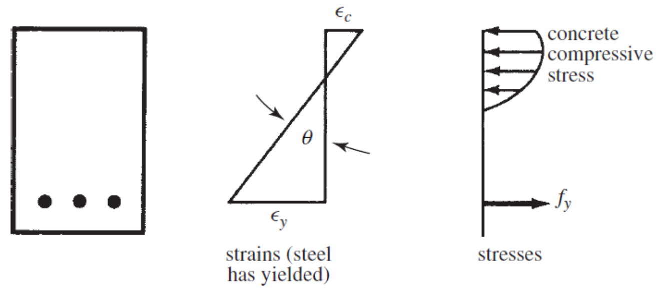


Uncracked concrete stage.

Stage 2: **Concrete Cracked–Elastic Stresses Stage:** As the load is increased after the modulus of rupture of the concrete is exceeded, cracks begin to develop in the bottom of the beam. The moment at which these cracks begin to form—that is, when the tensile stress in the bottom of the beam equals the modulus of rupture—is referred to as the *cracking moment*, M_{cr}. As the load is further increased, these cracks quickly spread up to the vicinity of the neutral axis, and then the neutral axis begins to move upward. This stage will continue as long as the compression stress in the top fibers is less than about one-half of the concrete's compression strength, f'_c , and as long as the steel stress is less than its yield stress. The straight-line stress–strain variation normally occurs in reinforced concrete beams under normal service-load conditions because at those loads, the stresses are generally less than 0.50 f'_c . To compute the concrete and steel stresses in this range, the transformed-area method is used. The *service* or *working* loads are the loads that are assumed to actually occur when a structure is in use or service. Under these loads, moments develop that are considerably larger than the cracking moments. Obviously, the tensile side of the beam will be cracked.



Stage 3: Beam Failure—Ultimate-Strength Stage: As the load is increased further so that the compressive stresses are greater than 0.50 f'_c, the tensile cracks move farther upward, as does the neutral axis, and the concrete compression stresses begin to change appreciably from a straight line. For this initial discussion, it is assumed that the reinforcing bars have yielded. The stress variation is much like below.

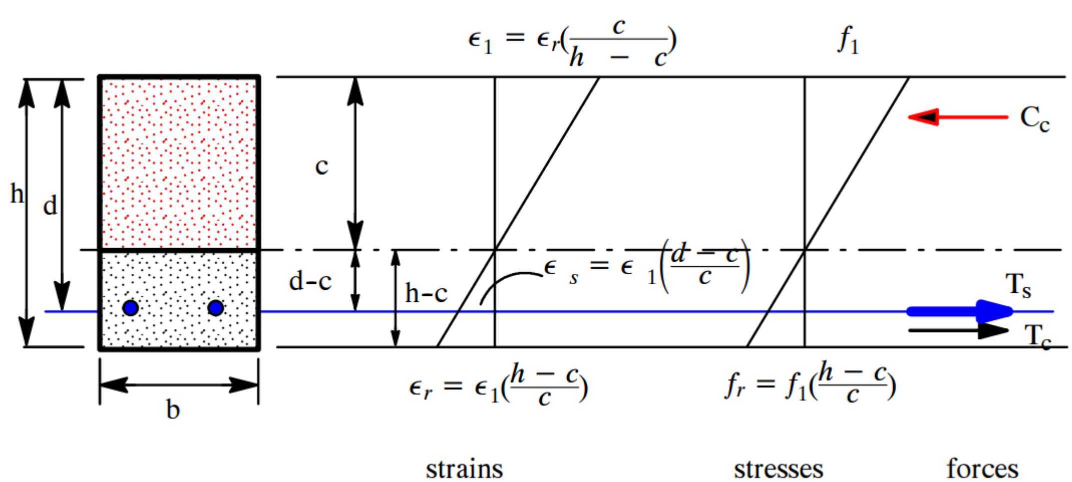


Ultimate-strength stage.

Reinforced Concrete Beam - Uncracked

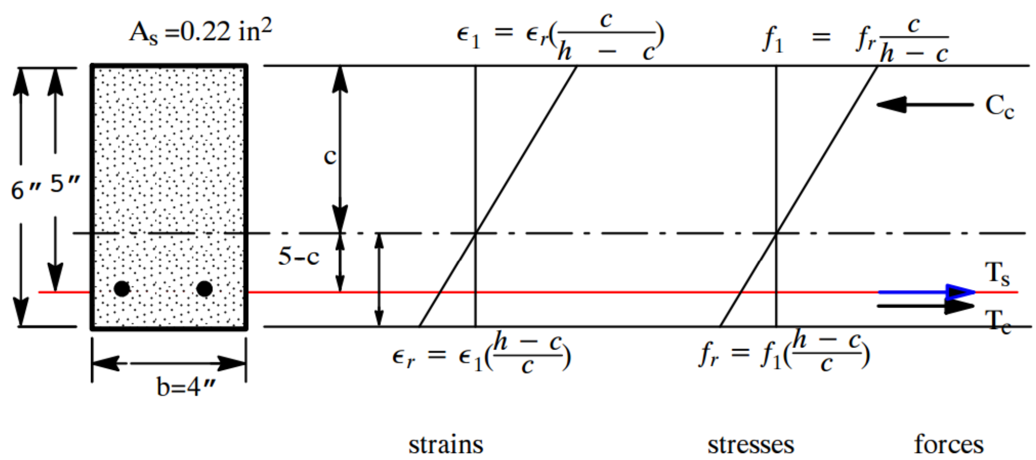
1. Assumptions:
   a. Strains vary linearly with distance from N.A.
   b. Linear stress-strain relationship;
   c. Strain compatibility between steel and concrete $\varepsilon = \varepsilon_s = \varepsilon_c$

2. Reinforced concrete beam. Before cracking



Example 1. Calculate Cracking Moment (M_{cr})

Calculate the moment of the section shown below when maximum tensile stress in concrete is equal to f_r (Cracking Moment)



Given Material Properties

f'_c = 3200 psi

f_r = 500 psi = 0.5 ksi

$$E_c = 57,000\sqrt{3200} = 3,220,000\ psi = 3,220\ ksi$$

Solution

$$\rho = \frac{A_s}{bd} = \frac{0.22(in^2)}{4(in) \times 5(in)} = 0.011$$

$$\frac{E_s}{E_c} = n = \frac{29,000\ (ksi)}{3,220\ (ksi)} = 9.01 \approx 9$$

$$\frac{h}{d} = \frac{6}{5} = 1.2$$

$$\frac{c}{d} = \frac{2\rho(n - 1) + (\frac{h}{d})^2}{2\rho(n - 1) + 2(\frac{h}{d})} = \frac{2 \times 0.011 \times 8 + (1.2)^2}{2 \times 0.011 \times 8 + 2 \times (1.2)} = 0.627$$

$$c = 0.627d = 0.627 \times 5(in) = 3.14\ inches$$

$$f_1 = f_r\left(\frac{c}{h - c}\right) = 0.5 \times \frac{3.14}{6 - 3.14} = 0.549 \ ksi$$

Having f_1, we can easily calculate all forces:

$$C_c = \frac{1}{2}f_1cb = \frac{1}{2} \times 0.549(ksi) \times 3.14(in) \times 4(in) = 3.45 \ kip$$

$$T_s = f_1\left(\frac{d - c}{c}\right)(n - 1)A_s$$

$$= 0.549(ksi) \times \frac{5(in) - 3.14(in)}{3.14(in)} \times (9 - 1) \times 0.22(in^2) = 0.57 \ kips$$

$$T_c = \frac{1}{2}f_1\left(\frac{h - c}{c}\right)(h - c)b$$

$$= \frac{1}{2} \times 0.549(ksi) \times \frac{[6(in) - 3.14(in)]^2}{3.14(in)} \times 4(in) = 2.86 \quad kips$$

Check equilibrium, does it satisfy $C_c = T_s + T_c$?

$C_c = 0.57 + 2.86 = 3.43$ kips $= C_c$ kips ; the difference is due to rounding error associated with calculating "c"

Calculate moment about N.A. (or any point on the cross section)

Force Kips	Moment Arm inches	Moment in-kips
$C_c = 3.45 \ kips$	$\frac{2}{3}c = \frac{2}{3} \times 3.14 = 2.09$	$= 7.21$
$T_s = 0.57$	$d - c = 5 - 3.14 = 1.86$	$= 1.06$
$T_c = 2.86$	$\frac{2}{3}(h - c) = \frac{2}{3}(6 - 3.14) = 1.91$	$= 5.46$

Total M = 13.73 in-kips