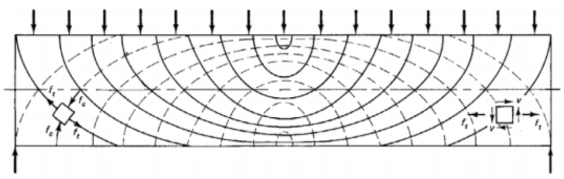
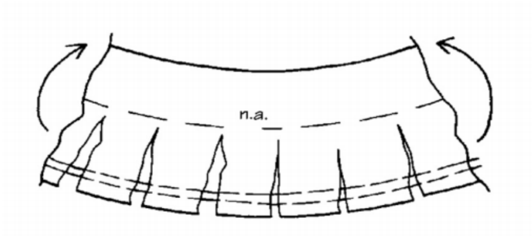
Design of Rectangular Beams
Beam proportions.
The most economical beam sections are usually obtained for shorter beams (up to 20 ft or 25 ft in length), when the ratio of *d* to *b* is in the range of 1.5 to 2.
For longer spans, better economy is usually obtained if deep, narrow sections are used. The depths may be as large as three or four times the widths.

The stress trajectories in this simple beam, show principle tension as solid lines.

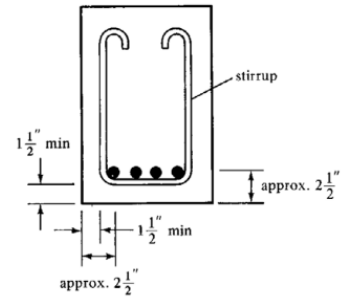


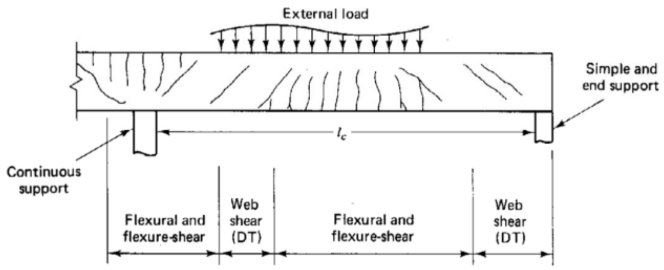
Reinforcement must be placed to resist these tensile forces

In beams continuous over supports, the stress reverses (negative moment).
In such areas, tensile steel is on top.

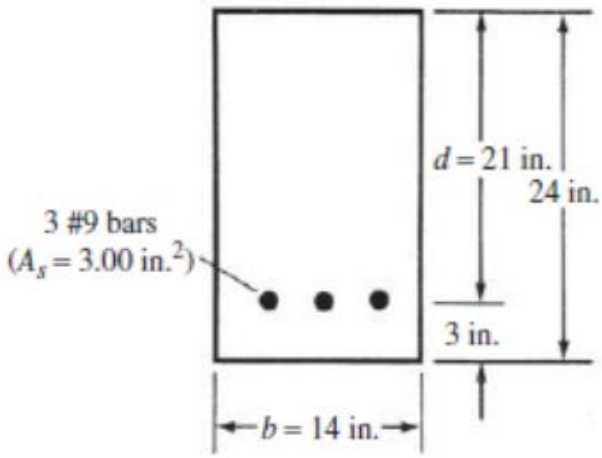


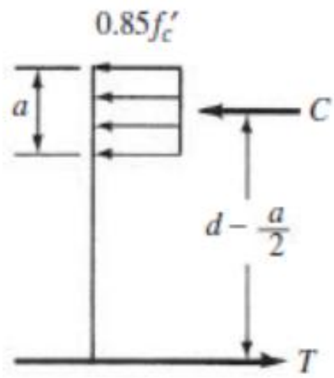
Shear reinforcement is provided by vertical or sloping stirrups.





Rectangular Beam:



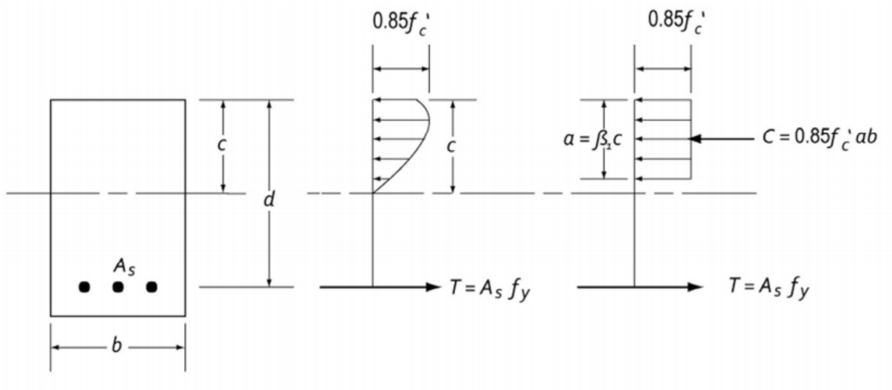


Beam section should be designed adequately to limit the deflection that affects the serviceability of structure adversely. According to ACI 318 section 9.5.2.1 minimum thickness of beams are provided in the following Table.

Table: Minimum thickness of nonprestressed beams.

	Simply supported	One end continuous	Both ends continuous	Cantilever
Minimum thickness	$\frac{l}{16}$	$\frac{l}{18.5}$	$\frac{l}{21}$	$\frac{l}{8}$

Flexure Equations



$$C = T$$

$$0.85f_c'ab = A_s f_y$$

solving for a,

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{\rho f_y d}{0.85 f_c'}$$

$$\rho = \frac{A_s}{bd}$$

$$M_n = T\left(d - \frac{a}{2}\right) = A_s f_y\left(d - \frac{a}{2}\right)$$

$$M_u = \phi M_n$$

$$M_u = \phi M_n = \phi A_s f_y\left(d - \frac{a}{2}\right)$$

$$M_u = \phi A_s f_y d\left(1 - 0.59\frac{\rho f_y}{f_c'}\right)$$

Design of beam example 1 (done by Mr. Naim Hassan)

- Design a simply supported beam of clear span 20 ft,
  Total depth of beam is 22 inches, given width 14”, live load 3k/ft ; given concrete compressive strength is 3 ksi and 60 grade steel.

  Solution:

  Beam self weight = $\frac{14\times22\times150}{144}$ lb/f t = 0.32 K/f t

  Total load, $w_u = 1.2 \times 0.32 + 1.6 \times 3$

$w_u = 5.184$ K/f t

$$M_n = \frac{w\,L^2}{8} = \frac{5.184\times 20^2}{8} = 260 \text{ K-f t}$$

d= 22”-2.5”= 19.5 in

$$A_s = \frac{M_n}{\emptyset f_y(d-\frac{a}{2})} = \frac{260\times 12}{0.9\times 60(19.5-\frac{5}{2})} = 3.398 \text{ in}^2$$ [Assume ,a=5 in]

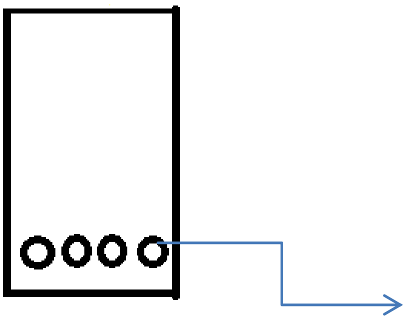
$$a= \frac{A_s f_y}{0.85 f''_c\ b} = \frac{3.398\times 60}{0.85\times 3\times 14} = 5.71 \text{ in}$$

$$A_s = \frac{M_n}{\emptyset f_y(d-\frac{a}{2})} = \frac{260\times 12}{0.9\times 60(19.5-\frac{5.71}{2})} = 3.47 \text{ in}^2$$

$$a= \frac{A_s f_y}{0.85 f''_c\ b} = \frac{3.47\times 60}{0.85\times 3\times 14} = 5.89 \text{ in}$$

$$A_s = \frac{M_n}{\emptyset f_y(d-\frac{a}{2})} = \frac{260\times 12}{0.9\times 60(19.5-\frac{5.89}{2})} = 3.48 \text{ in}^2$$

A_s= 3.48 in^2



4#9 bar

Example 2 – Flexural Design

Given: A simply supported, reinforced concrete beam with an overall depth of 16 in., an effective depth of 13.5 in., and a width of 12 in. is reinforced with Grade 60 bars and has normal weight concrete with a compressive strength of 3000 psi. Determine the area of reinforcement required for the beam to carry a superimposed live load of 1.1 kips/ft on an effective span of 20 ft.

The weight of the beam is w_D

$= 16$ in. x 12 in. x 12 in. x 150 pcf / 1728 in^3/ft^3 = 200 lb/ft

Moments:

$M_D = 1/8\, w_D\, l^2 = (1/8)(0.2 \text{ kips/ft})(20 \text{ ft})^2 = 10$ kip-ft

$M_L = 1/8\, w_L\, l^2 = (1/8)(1.1 \text{ kips/ft})(20 \text{ ft})^2 = 55$ kip-ft

and the factored total moment $M_U = 1.2(10) + 1.6(55) = 100$ kip-ft

For design: $\Phi M_n \geq M_u$

or $M_{n,req'd} = M_u / \Phi$

By trial and error:

- choose $a = 4.0$ in.
- $A_{s,req'd}$ = [(100 kip-ft x 12 in./ft)/0.9] / [60 ksi (13.5 in. – 4.0 in./2)]
  = 1.93 in^2
- new a = [1.93 in^2 x 60 ksi] / [0.85 x 3 ksi x 12 in.] = 3.78 in.
- new $A_{s,req'd}$ with a of 3.78 in. = 1.89 in^2 ⇨ close enough

According to ACI 318 section 10.5 minimum tensile reinforcement should be provided to resist the cracking moment. For any section minimum reinforcement can be calculated by the equation-

$$(A_s)_{min} = \text{Larger of } \frac{3\sqrt{f_c'}}{f_y} b_w d \;;\; \frac{200}{f_y} b_w d, \qquad \text{where } f_c' \text{ and } f_y \text{ are in psi}$$

$A_{s,max} = \rho_{max} * (b_w d)$; [(0.85)* (0.85)*(3/60)] x [3/7] *(12* 13.5] = 2.50 in^2

$$\rho_{max} = 0.85\, \beta_1\, (f'_c / f_y) \left(\frac{0.003}{0.003+0.004}\right) = 0.85 \times (0.85) \times (f'_c / f_y) \times (3/7)$$

Check max and min reinforcement values:

$A_{s,max}$ = [(0.85)(3 ksi)(0.85)(12 in.)/(60 ksi)] x [(3)(13.5)/(7)] = 2.50 in^2

$A_{s,min}$ = larger of [(3) x sqrt(3000 psi)(12 in.)(13.5 in.)]/(60,000 psi) = 0.44 in^2

or [(200 psi)(12 in.)(13.5 in.)] / (60,000 psi) = 0.54 in^2 (controls)

So $A_{s,req'd}$ = 1.89 in^2 meets max and min values

Choose 2 No. 9 bars giving A_s, provided = 2.0 in^2

If "d" or "h" of beam is not given then: Find "d" required.

$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + 0.004}$

$\rho_{max} = 0.85\ \beta_1\ (f'_c/\ f_y)\ (\frac{0.003}{0.003+0.004}) = 0.85\times(0.85) \times (f'_c/\ f_y)\times(3/7)$

Example: $A_{s,max} = \rho_{max} * (b_w d)$;

[(0.85)* (0.85)*(3/60)] x [3/7] *(12* 13.5] = 2.50 in^2

Assume $\rho = 0.5\times \rho_{max}$ or, $\rho = \rho_{max}$

$d^2 = \frac{Mu}{\phi \rho\ fy\ b\ (1 - 0.59\ \rho\left(\frac{fy}{f'c}\right))}$

Find "d" required.

Assume "a". Find "A_s"

$A_s = \frac{Mu}{\emptyset fy(d - \frac{a}{2})}$

Checking the assumed depth, a

$a = Asfy/(0.85 f'c\ b)$

Iterate to find A_s.