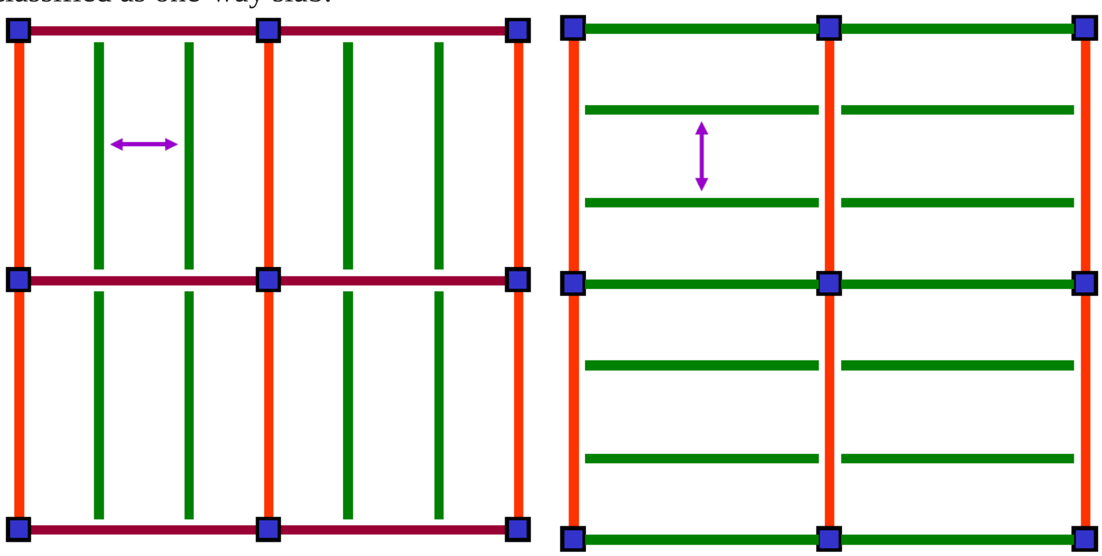
Notes on One Way Slab Design

-Dr. E. R. Latifee

Question: Define the term slab, one way slab, two way slab, Shrinkage and temperature reinforcement, main reinforcement.

Definition:
Slab is a common term used to describe the structural elements such as floors and roofs where the thickness of the element is very small compared to the length and width.
Rectangular Slabs are mainly two types: one-way and two-way.
One-way slab means the load is transferred in one direction, therefore, the bending occurs in one direction only. When a rectangular slab is supported on all four sides, but the ratio of longer side, L, to shorter side, S, is two or more, $L/S \geq 2.0$, then, the slab will act as a one-way slab, **with bending primarily occurring in the short direction**. Such slabs are designed as one-way slabs where the span length (L_n) is equal to the length of short direction.
Here, main reinforcement is placed in the shorter direction which is the span, while shrinkage reinforcement is provided in the longer direction to limit cracking.
Note: Special case: When the slab is supported on two sides only, which is uncommon, the load will be transferred to these sides regardless of its longer span to shorter span ratio, and it will be classified as one-way slab.



Two-way slab means the load is transferred in two directions, therefore, the bending occurs in two directions. In this case, L/S is less than 2.0.

Slab reinforcement: Main reinforcement: Used to resist bending moment due to Live and dead load.

Shrinkage and temperature reinforcement: Used to prevent cracking from shrinkage/contraction of the concrete due to water loss from concrete mass at different stages, temperature variation and differential thermal gradient.

What are the assumptions for flexure theory in beam design?

There are three basic assumptions for flexure

I. Sections perpendicular to the axis of bending which are plane before bending remain plane after bending.
II. The strain is in the reinforcement is equal to the strain in the concrete at the same level.
III. The stresses in the concrete and reinforcement can be computed from the strains using stress- strain curves for concrete and steel.

Additional

IV. The tensile strength of concrete is neglected in flexural strength calculations (ACI sec. 10.2.5).
V. Concrete is assumed to fail when the compressive strain reaches a limiting value .

VI. The compressive stress strain relationship for concrete may be assumed to be rectangular, trapezoidal, parabolic or any other shape that results in prediction of strength in substantial agreement with results of comprehensive tests (ACI 10.2.6).

What does it mean by beam design and beam analysis?

Design: Given a factored load effect such as Mu, select a suitable cross section, including dimensions, concrete strength, reinforcement detail and others.

Analysis: Given a cross section, concrete strength, reinforcement size, location and yield strength, compute the resistance or capacity.

ACI 318 -11- *Building Code Requirements for Structural Concrete- by American Concrete Institute*, ACI.

Minimum Thickness of One-way Slabs (ACI Code 9.5.2) (ACI 318-11)

These values are applicable for normal loading conditions and for slabs not supporting or attached to partitions or other construction likely to be damaged by large deflections.

Simple span solid slabs*	$ln/_{20}$
Cantilever	$ln/_{10}$
Solid Slabs Continuous at one End	$ln/_{24}$
Solid Slabs Continuous at both Ends	$ln/_{28}$

*Minimum thickness for cantilevers can be considered equal to twice that of for a simple span

Where, l_n is the span length in the direction of bending.

What is positive moment and negative moment and reinforcement regarding beam?

Answer: A moment that causes compression on top surface of a beam and tension on the bottom surface will be called a positive moment. A moment that causes tension on top surface of a beam and compression on the bottom surface will be called a negative moment. The steel provided in the positive moment region is called positive reinforcement and the steel provided in the negative moment region is called negative reinforcement.

Q. Explain Tension, Compression & Balanced Failure:

Depending upon the properties of a beam, flexural failure may occur in three different ways:

1. Tension failure: Reinforcement yields before concrete crushes. (reaches its limiting compressive strain). Such a beam is said to be under-reinforced.
2. Compression failure: Concrete crushes before steel yields. Such a beam is said to be over-reinforced
3. Balanced failure: Concrete crushes (**crushing strain)** and **steel yields (yield strain)** simultaneously. Such a beam has balanced reinforcement.

$$\rho_{\text{balance}=}(0.85)\ \beta_1 * \frac{f'c}{fy} * \frac{\epsilon cu}{\epsilon cu+\epsilon y}$$

Substitute, ε_{cu} = 0.003 concrete compressive strain at failure,

E_s=Modulus of elasticity of steel=29000ksi,

Steel yield strain ε_y= stress/E_s = f_y/29,000,000 psi

Now, substituting,

$$\rho_{\text{balance}=}(0.85)\ \beta_1 * \frac{f'c}{fy} * \frac{87000}{87000+fy}$$

β_1= 0.85 for $f'c \leq$ 4000psi

Design of a one-way slab

Step-1

Selection of thickness, h

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Where, l_n is the span length in the direction of bending.

For the negative moment at the interior face of the exterior support , for positive moment , and for shear , ln =clear span of the span in question.

For negative moment at interior supports , ln = average of the clear spans of the adjacent spans.

(ACI Code 9.5.2 provided that slabs are of normal-weight concrete (wc=145 pcf) using Grade 60 reinforcement)

The total slab thickness h is usually rounded to the next higher ¼ inch for slabs up to 6 inch thickness and to the next higher ½ inch for thicker slabs. Best economy is often achieved when the slab thickness is selected to match nominal lumber dimensions.

The concrete protection below the reinforcement should follow the requirements of ACI code 7.7.1 calling for ¾ inch below the bottom of the steel. In a typical slab 1 inch below the centre of the steel may be assumed.

Step-2

Calculation of load

Two types of load-Dead load & Live load

a) Dead load calculation=

Self weight calculation=$\frac{thickness}{(12)}$ *unit weight of concrete; where thickness is in inches

b) Live load calculation, assume

40 psf (residential), 100 psf (stair, lobby, public places etc.)

Note: 40 psf = 1.92 kN/m^2 , 100 psf = 4.79 kN/m^2

BNBC Live Load:

Table 2.3.1 Minimum Uniformly Distributed Live Loads, And Minimum Concentrated Live Loads (Contd.)

Penal Institutions		
Cell blocks	1.92	
Corridors	4.79	
Residential		
Dwellings (one- and two-family)		
Uninhabitable attics without storage	0.48	
Uninhabitable attics with storage	0.96	
Habitable attics and sleeping areas	1.44	
All other areas except stairs and balconies	1.92	
Hotels and multifamily houses		
Private rooms and corridors serving them	1.92	
Public rooms and corridors serving them	4.79	

Step-3

Determination of factored load

Factored dead load =calculated dead load*1.2

Factored live load = calculated live load*1.6

Calculate the factored load by magnifying service dead and live loads according to this equation $w_u = 1.20*DL + 1.60LL$. The dead load includes own weight of the slab.

Step-4

Determination of factored moment: **Moments for beams and one way slab**

Factored moment coefficient found in ACI Code section 8.3.3

Q. What are the criteria for using ACI moment coefficients for non pre-stressed continuous beams or one way slabs?

ACI MOMENT COEFFICIENT

Because the calculations necessary to derive the moment are tedious, approximate moment coefficients are presented in ACI Sec 8.3.3 which can be used to calculate the moment and shear envelopes for non pre-stressed continuous beams or one way slabs that meet the following criteria:

1. There are two or more spans
2. The spans are approximately equal with the longer of two adjacent spans not more than 1.2 times the Length of the shorter one
3. The loads are uniformly distributed.
4. The unit live load does not exceed three times the unit dead load. The word 'unit' means the unfactored load per foot.
5. The beams must be prismatic.
6. The beam must be in a braced frame without significant moments due to lateral loads(not stated in ACI code but necessary for the coefficients to apply).

The maximum positive and negative moments and shears are computed from the following equations: $\mathbf{M_{u=}C_m(w_u l_n^2)}$

The *ACI Code318-11 section 8.3.3* gives the moments at mid spans and at faces of supports, as follows.

Positive moment

End spans
Discontinuous end unrestrained …….. $w_u l_n^2/ 11$
Discontinuous end integral with support……… $w_u l_n^2/ 14$
Interior span…………………. $w_u l_n^2/ 16$

Negative moments at exterior face of first interior support
Two spans……. $w_u l_n^2/ 9$
More than two spans……. $w_u l_n^2/ 10$

Negative moment

Negative moment at other faces of interior supports……. $w_u l_n^2/ 11$
Negative moment at face of all supports for
Slabs with spans not exceeding 10 ft; and beams where ratio of sum of column stiffness to beam stiffness exceeds eight at each end of the span……. w_u

Negative moment at interior face of exterior support for members built integrally with supports
Where support is spandrel beam ……. $w_u l_n^2/ 2$
Where support is column ……. $w_u l_n^2/ 16$
Shear in end members at face of first interior support V_u…… $1.15\ w_u l_n/ 2$
Shear at face of all other supports, V_u …… $w_u l_n/ 2$

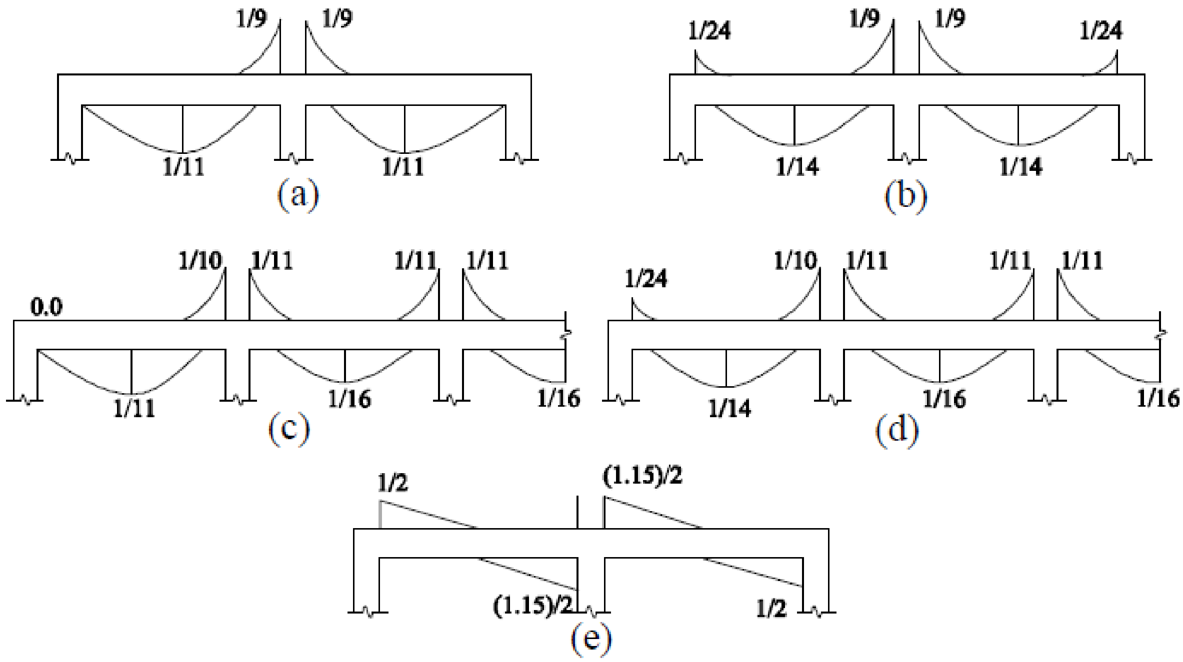


Figure : (a) Two spans, exterior edge unrestrained; (b) two spans, support is spandrel beam; (c) more than two spans, exterior edge unrestrained; (d) more than two spans, support is spandrel beam; (e) two spans, shearing force diagram

Step -5

Determination of maximum reinforcement ratio, ρ_{max},. to ensure underreinforced behavior

$$\rho_{max} = (.85)\ \beta_1 * \frac{f'c}{fy} * \frac{\epsilon u}{\epsilon u + \epsilon t}$$

10.3.5 — For nonprestressed flexural members and nonprestressed members with factored axial compressive load less than $\mathbf{0.10} f'_c A_g$, ε_t at nominal strength shall not be less than 0.004.

R10.3.5 — The effect of this limitation is to restrict the reinforcement ratio in nonprestressed beams to about the same ratio as in editions of the Code before 2002. The reinforcement limit of $\mathbf{0.75}\rho_b$ results in a net tensile strain in extreme tension steel at nominal strength of 0.00376. The limit of 0.004 is slightly more conservative. This limitation does not apply to prestressed members.

Example:

$$\rho_{max} = 0.85\ \beta_1\ (f'_c / f_y)\ \left(\frac{0.003}{0.003+0.004}\right) = (0.85)^2\ .(4/60).\ \frac{0.003}{0.003+0.004} = 0.02064$$

Where,

f'_c = specified compressive strength of concrete cylinder (4" by 8") at 28 days

fy = yield strength of steel

ε_u=compressive strain of concrete (observed failure at strain 0.003 to 0.004), take 0.003,

ε_t= minimum net tensile strain in steel, take 0.004 to ensure underreinforced behavior, i.e., tension failure.

$$\rho_{balance}=(0.85)\ \beta_1 * \frac{f'c}{fy} * \frac{87000}{87000+fy} = (0.85)^2\ .(4/60).\ \frac{87000}{87000+60000} = 0.0285$$

Limit: $0.75* \rho_{balance} = 0.75*0.0285 = 0.2138$

$\rho_{max} < 0.75 * \rho_{balance}$ (OK, tension controlled, underreinforced)

Step-6

Assume $\rho = 0.5\times \rho_{max}$ or, $\rho = \rho_{max}$

Check of effective depth, d

$$d^2 = \frac{Mu}{\phi\rho\ fy\ b\ (1-0.59\ \rho\left(\frac{fy}{f'c}\right))}$$

where, d= effective depth of the slab

d = distance from extreme compression fiber to centroid of longitudinal tension reinforcement, in., Chapters 7, 9-12, 14, 17, 18, 21,

M_u=factored moment

Strength reduction factor, $\emptyset$= 0.9

Bangladesh National Building Code 2011

8.1.7.2 **Cast-in-place Concrete :**

(a) Minimum concrete cover for concrete cast against and permanently exposed to earth shall be 75 mm.

(b) Concrete exposed to earth or weather:

	Minimum cover, mm
19 mm ϕ through 57 mm ϕ	50
16 mm ϕ bar and smaller	40

(c) The following minimum concrete cover may be provided for reinforcement for concrete surfaces not exposed to weather or in contact with ground:

	Minimum cover, mm
Slabs, Walls :	
40 mm ϕ to 57 mm ϕ	40
36 mm ϕ bar and smaller	20

	Minimum cover, mm
Beams, Columns :	
Primary reinforcement, Ties, stirrups, spirals	40

Thickness, h_1 = d, effective thickness + ½ bar dia.+ 20 mm or ¾ inches

Minimum Thickness of One-way Slabs (ACI Code 9.5.2) (ACI 318-11) –Deflection criterion

Say $h_2 = l_n/24$ or 28 etc., see table below.

These values are applicable for normal loading conditions and for slabs not supporting or attached to partitions or other construction likely to be damaged by large deflections.

Simple span solid slabs*	$ln/_{20}$
Cantilever	$ln/_{10}$
Solid Slabs Continuous at one End	$ln/_{24}$
Solid Slabs Continuous at both Ends	$ln/_{28}$

Take the higher thickness between h_1 and h_2 = required h

Then, d = h – 20 mm - ½ bar dia.

Step-7

Determination of steel area, A_s

$A_s = \frac{Mu}{\emptyset fy(d-\frac{a}{2})}$

Checking the assumed depth, a

$a = Asfy/(0.85f'c\ b)$

Iterate to find A_s.

Minimum reinforcement area for slab in both short and long direction is equal to the Shrinkage and temperature reinforcement.

For slab - $A_{s,\ minimum} = A_{s,\ (S\&T)}$

Minimum Reinforcement Ratio for Shrinkage and Temperature(S&T) Reinforcement:
ACI Code 7.12.2.1 specifies the minimum ratios of Shrinkage and Temperature (S&T) Reinforcement area to gross concrete area (i.e. based on the), $\rho_{minimum=}$ A_s/bh; b = width of strip, such as 12 inches, h = total depth of the slab, inches, but no less than 0.0014

Type of reinforcement used	Minimum ratio of steel area to gross concrete area
Slabs where Grade 40 or Grade 50 deformed bars are used	0.0020
Slabs where Grade 60 deformed bars or welded wire fabric (smooth or deformed) are used	0.0018
Slabs where reinforcement with yield strength exceeding 60,000psi measured at yield strain of 0.35 percent is used	$\frac{0.0018 \times 60{,}000}{fy}$

Reinforcement for shrinkage and temperature stresses normal to the principal reinforcement should be provided in a structural slab in which the principal reinforcement extends in one direction only.

Step-8

Determining the spacing of the steel bars

Spacing=$\frac{area\ of\ the\ bar\ used * width\ of\ the\ strip}{required\ steel\ area}$

Maximum spacing, S, for Main reinforcement for flexure, with slab thickness, h-

ACI 10.5.4: S ≤ smaller of (3 h or 18 in.)

Bar diameter	Area (mm^2)
8 mm	50.3
10 mm	78.5
12 mm	113.1
14 mm	153.86

In one way slab since all of the load on the slab must be transmitted to the two supporting beams, it follows that all of the reinforcement should be placed at right angles to these beams, with the exception of any bars that may be placed in the other direction to control shrinkage and temperature cracking.

The lateral spacing of the bars, except those used to control shrinkage and temperature cracks should not exceed 3 times the thickness h or 18 inch, whichever is less, according to ACI Code 7.6.5.

Generally, bar size should be selected so that the actual spacing is not less than about 1.5 times the slab thickness, to avoid excessive cost for bar fabrication & handling. Also to reduce cost, straight bars are usually used for slab reinforcement.

Spacing for Shrinkage and Temperature (S&T) Reinforcement: „
ACI 7.12.2.2 S_{max} (S&T) = min (5h or 18 in.)

Example 2.1: One way slab design:

A reinforced concrete slab is to be built integrally with its supports and consists of two equal spans, each with a clear span of 15 ft. The service live load is 40 psf and 4000 psi concrete is specified with 60 grade steel (yield stress equal to 60,000 psi).

Solution:

Minimum thickness $l_n/28 = 15 \text{ X } 12/28 = 6.43$ in.≈ 6.50 in.

Self-weight of slab = 150 X 6.50/12 = 81 psf [unit weight of concrete 150 Ib/ft^3]

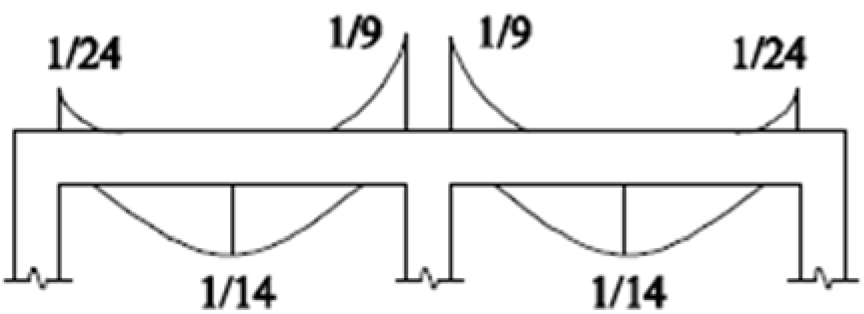
The specified live load and computed dead load are multiplied by the Load factors:

Dead load = 81 × 1.2 = 97 psf

Live load = 40 × 1.6 = 64 psf

Total load = 161 psf

Using ACI moment coefficient,



At support: -M = 1/9 X 0.161 X 15^2 = 4.025 ft-kip

At mid-span: +M = 1/14 X 0.161 X 15^2 = 2.5875 ft-kip

At exterior support: -M = 1/24 X 0.161 X 15^2 = 1.51 ft-kip

$$\rho_{max} = 0.85\ \beta_1\ f'_c /\ f_y \frac{0.003}{0.003+0.004} = (0.85)^2\ 4/60\ \frac{0.003}{0.003+0.004} = 0.021$$

Minimum effective depth = (6.50-1) in. = 5.50 in.
[Note: minimum clear cover 20 mm or 0.75 inches for slab + ½ bar diameter ≈ 1 inch.]

Minimum required effective depth would be found from the equation

Now, we need to work only with one "d check" with Maximum moment,

since all through the slab, the same thickness will be given.

$$d^2 = \frac{Mu}{\phi\rho\ fy\ b\ (\ 1-0.59\ \rho\left(\frac{fy}{f'c}\right))}$$

$$= \frac{4.025\ X\ 12}{0.90\ X\ 0.021\ X\ 60\ X\ 12\ X\ (1-0.59\ X\ 0.021\ X\left(\frac{60}{4}\right))} = 4.36 \text{ in}^2$$

Now, d = 2.08 in. ≈ 2.1 in.< minimum 5.50 in

Therefore, deflection controls for depth of the slab.

Final slab thickness = 6.50 in.
Assume, stress block depth a = 1 in.
The area of steel required per foot width in the top of the slab is

$A_s = \frac{Mu}{\phi\ fy\ (d-\left(\frac{a}{2}\right))} = \frac{4.025\ X12}{0.90\ X\ 60\ X\ (5.50-0.5)} = 0.178 \text{ in}^2$
Checking a.
$a = \frac{As\ fy}{0.85\ f'c\ b} = \frac{0.178\ X60}{0.85\ X\ 4\ X\ 12} = 0.2617$ in.
Trail 2:
a = 0.26 in.
$A_s = \frac{Mu}{\phi\ fy\ (d-\left(\frac{a}{2}\right))} = \frac{4.025\ X12}{0.90\ X\ 60\ X\ (5.50-0.13)} = 0.166 \text{ in}^2$
Again checking a,
$a = \frac{As\ fy}{0.85\ f'c\ b} = \frac{0.166\ X60}{0.85\ X\ 4\ X\ 12} = 0.245$ in. ≈ 0.26 in. (ok)

Minimum rebar for temperature and shrinkage, $A_{s,\ min}$ = 0.0018*12*6.5 = 0.1404 in^2/ft

Negative Steel required,

A_s negative for interior support, $A_s = \frac{Mu}{\phi\ fy\ (d-\left(\frac{a}{2}\right))} = \frac{4.025\ X12}{0.90\ X\ 60\ X\ (5.50-0.125)} = 0.166 \text{ in}^2$/ft > $A_{s,\ min}$, OK

A_s negative at exterior support, As = $\frac{1.51\ X12}{0.90\ X\ 60\ X\ (5.50-0.125)} = 0.06 \text{ in}^2$/ft0. <As, min, use $A_{s,\ min}$

Positive A_s, at mid span, $A_s = \frac{2.588\ X12}{0.90\ X\ 60\ X\ (5.50-0.125)} = 0.107 \text{ in}^2$/ft, <As, min, use $A_{s,\ min}$

Spacing=$\frac{area\ of\ the\ bar\ used * width\ of\ the\ strip}{required\ steel\ area}$

S_{max} for main reinforcement
ACI 10.5.4: S_{max} ≤ smaller of (3 h or 18 in.) =3*6.5 or 18, 18 inches governs
Shrinkage and temperature,
ACI 7.12.2.2 S_{max} (S&T) = min (5h or 18 in.), so 18 inches governs

Spacing, Let # 4 bar, (Area of steel 0.2 in^2)
A_s negative for interior support
S =0.2 *12/0.166 = 14.4 in, use 12 inches<Smax, OK

+ve Main rebar in short direction, S_{main} = 0.2 *12/0.1404 = 17.09 in, use 16 inches<Smax, OK

Shrinkage and temperature, Spacing needed = $S_{S\&T}$ = 0.2 *12/0.14= 17.1, use 16 inches<Smax, OK

Position	Bar size	Spacing (inches)
Main rebar- A_s positive short direction positive Moment-Bottom Bars	#4	16
Main rebar A_s negative at exterior support -Top Bars	#4	16
Main rebar A_s negative for interior support-Top Bars	#4	12
Shrinkage and temperature (S&T) reinforcement.-Long direction. Note that the depth within the slab is not critical for S&T steel. However, these are placed on top of main bottom bars below top bars.	#4	16

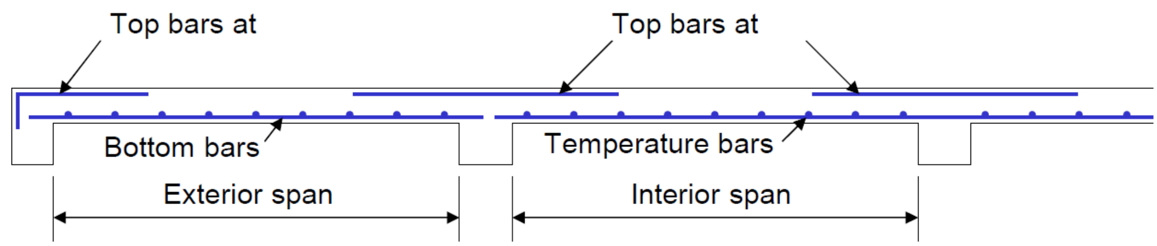
Shear Check:

Vu =(1.15×w_u×L_n)/2 = (1.15×0.161×15)/2 =1.388 kip

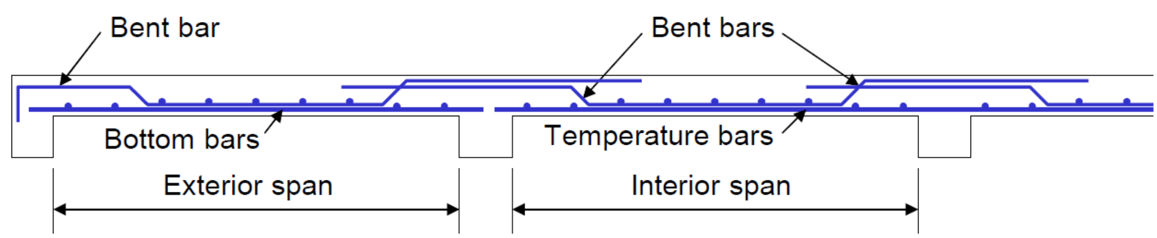
$$V_c = 2\sqrt{f_c'}\, b_w d$$

ϕVc = 0.85*2*(4000^0.5)*12*5.5/1000 =7.09 kip

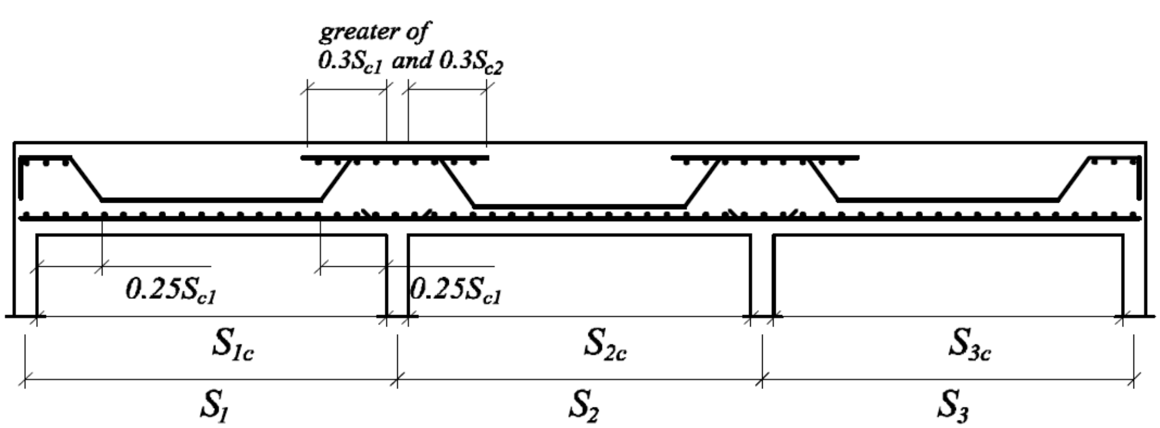
ϕVc ≥ Vu, therefore OK.

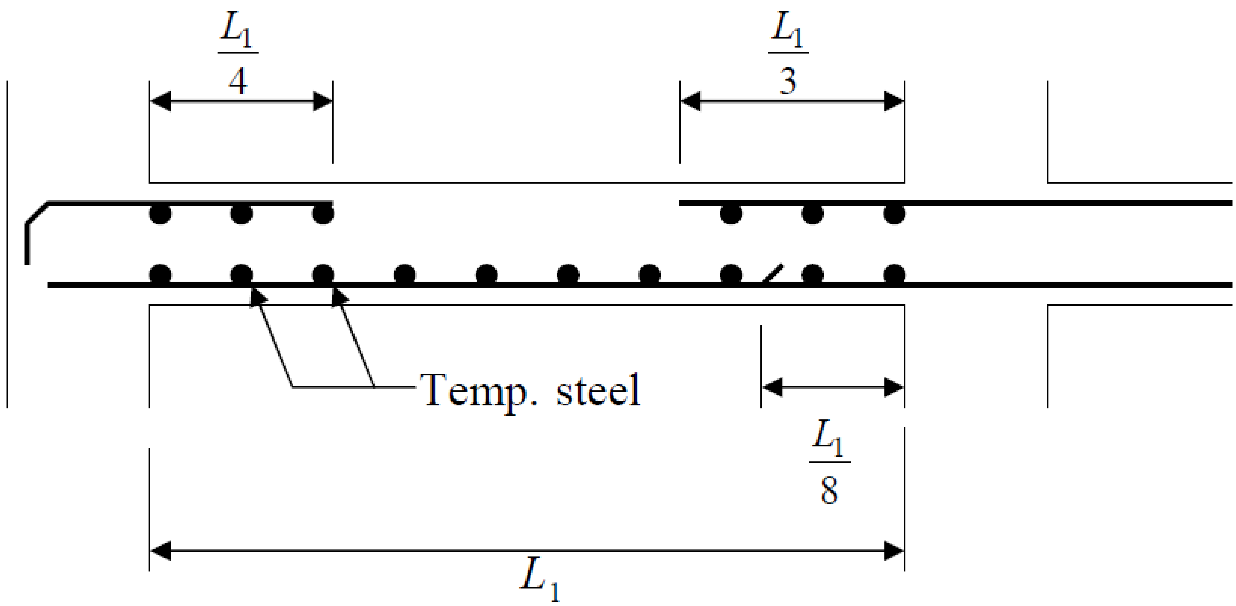


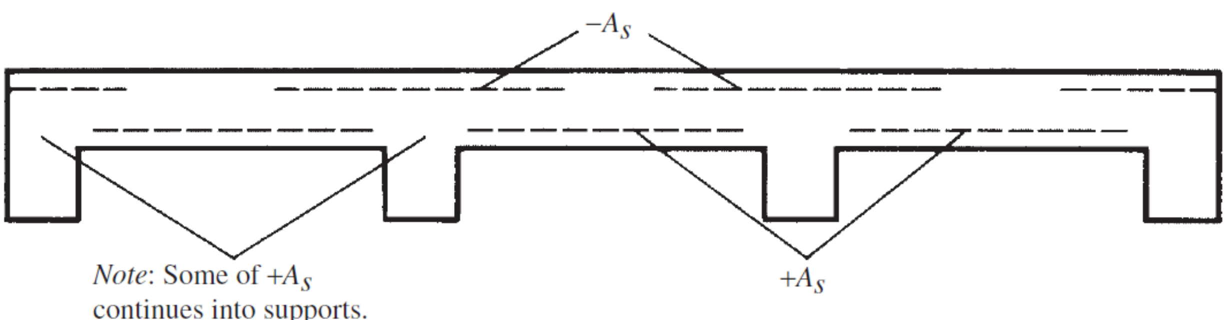
(a) Straight top and bottom bars



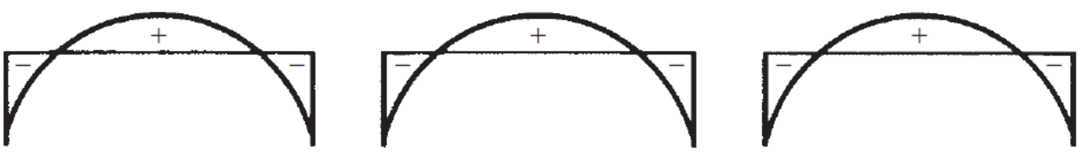
(b) Alternate straight and bent bars







(a)

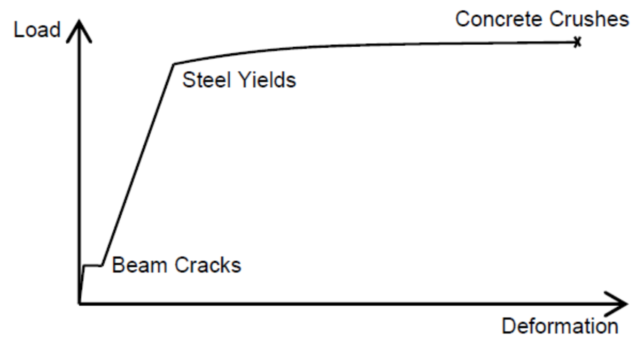


(b)

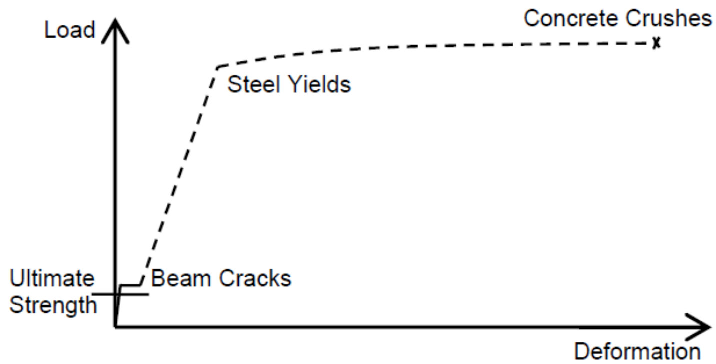
Position	Bar size	Spacing (inches)
Main rebar- short direction positive Moment-Bottom Bars	#4	10
Main rebar- short direction negative Moment-Top Bars	#4	10
Shrinkage and temperature (S&T) reinforcement.-Long direction. Note that the depth within the slab is not critical for S&T steel. However, these are placed on top of main bottom bars below top bars.	#4	16

Additional Notes:

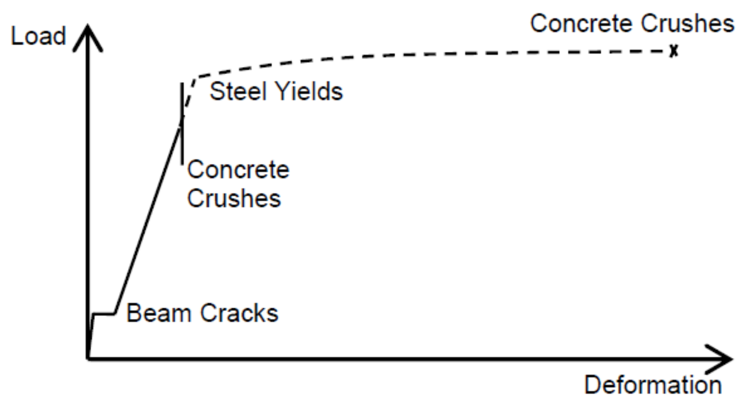
Failure Scenario #1 (just right steel). The preferred order of flexure response events is: beam cracks, steel yields, concrete crushes



Failure Scenario #2 (too little steel).
Mu < Mcr. In this case, the beam has so little steel, that when the beam does crack, it collapses. ACI 10.5.4 specifies a minimum area of steel , Asmin. For slabs, this is set equal to the reinforcement required to prevent excessive temperature and shrinkage cracking (ACI 7.12.2)



Failure Scenarios #3 (too much steel). es >> ey. In this case, the beam has so much steel that even at ultimate conditions (ec = 0.003) the steel has not yielded. ACI 10.3.5 specifies that the strain in the extreme tension steel must be well beyond yield (et > 0.004) to prevent crushing the concrete before observable deformation.

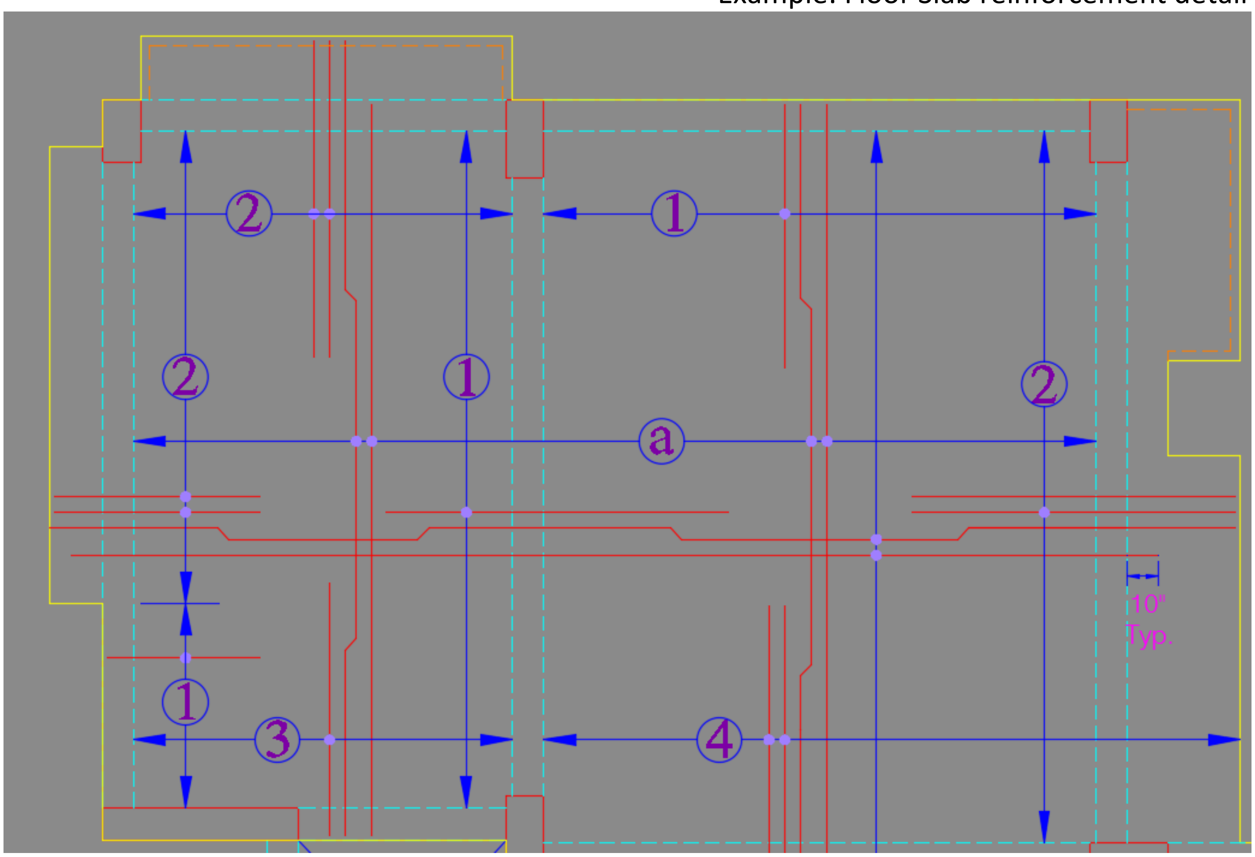


Distribute reinforcement. Stress transfer from the rebar to the surrounding concrete is more effective if smaller-diameter closely-spaced rebar is used, rather than larger-diameter, widely-spaced rebar.

Determine slab thickness. Slab thickness is usually controlled by deflection considerations or fire ratings. ACI sets minimum thicknesses for beams and one-way slabs *not supporting partitions likely to be damaged by deflections* unless deflections are calculated in ACI Table 9.5a. Another consideration when selecting a slab thickness is fire rating. For example, the minimum slab thickness for a two-hour fire rating is 4.75 inches. Finally, since shear reinforcement is not used in one-way slabs, the concrete must be able to resist the maximum shear force.

U.S. rebar size chart							
Imperial Bar Size	"Soft" Metric Size	Mass per unit length		Nominal Diameter		Nominal Area	
		lb/ft	(kg/m)	(inch)	(mm)	($inch^2$)	(mm^2)
#2	#6	0.167	0.249	0.250 = ¼	6.35	0.05	32
#3	#10	0.376	0.561	0.375 = ⅜	9.525	0.11	71
#4	#13	0.668	0.996	0.500 = ½	12.7	0.20	129
#5	#16	1.043	1.556	0.625 = ⅝	15.875	0.31	200
#6	#19	1.502	2.24	0.750 = ¾	19.05	0.44	284
#7	#22	2.044	3.049	0.875 = ⅞	22.225	0.60	387
#8	#25	2.670	3.982	1.000	25.4	0.79	509
#9	#29	3.400	5.071	1.128	28.65	1.00	645

Example: Floor Slab reinforcement detail



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