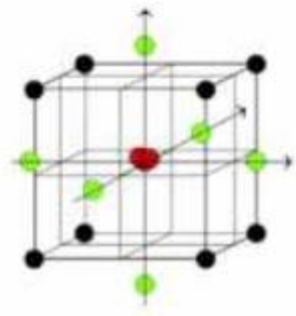
Introduction to Design of Experiments (DoE)



-Dr. E. R. Latifee

What is an Experiment?

A test done in order to learn something or to discover if something works or is true (Source: Cambridge dictionary).
(https://dictionary.cambridge.org/dictionary/english/experiment)

An experiment is a scientific test which is done in order to discover what happens to something in particular conditions.
(https://www.collinsdictionary.com/dictionary/english/experiment)

What is Design of experiments?

Design of experiments (DOE) is a method used to determine which variable(s) from a collection from variables is the most effective contributor to some process.
From: General Aviation Aircraft Design, 2014

Design of experiments, referred to as DOE, is a systematic approach to understanding how process and product parameters affect response variables such as processability, physical properties, or product performance.
Design of Experiments
John R. WagnerJr., ... Harold F. GilesJr., in Extrusion (Second Edition), 2014

What is factorial design of experiments?

Factorial Design: Factorial design is a useful technique to investigate main and interaction effects of the variables chosen in any design of experiment.
From: Nanotechnology in Eco-efficient Construction (Second Edition), 2019

2^k experiments: In a factorial design, the influences of all experimental independent variables (known as factors), and interaction effects on the response or responses are investigated. If the combinations of k factors are investigated at two levels, a factorial design will consist of 2^k experiments. It is expressed as *level factor*. For example, if 3 factors are investigated at 2 levels, this will need 2^3 =8 experiments.
The response (Y), dependent on factors A, B, C can be expressed as;
Y= f (A, B, C).

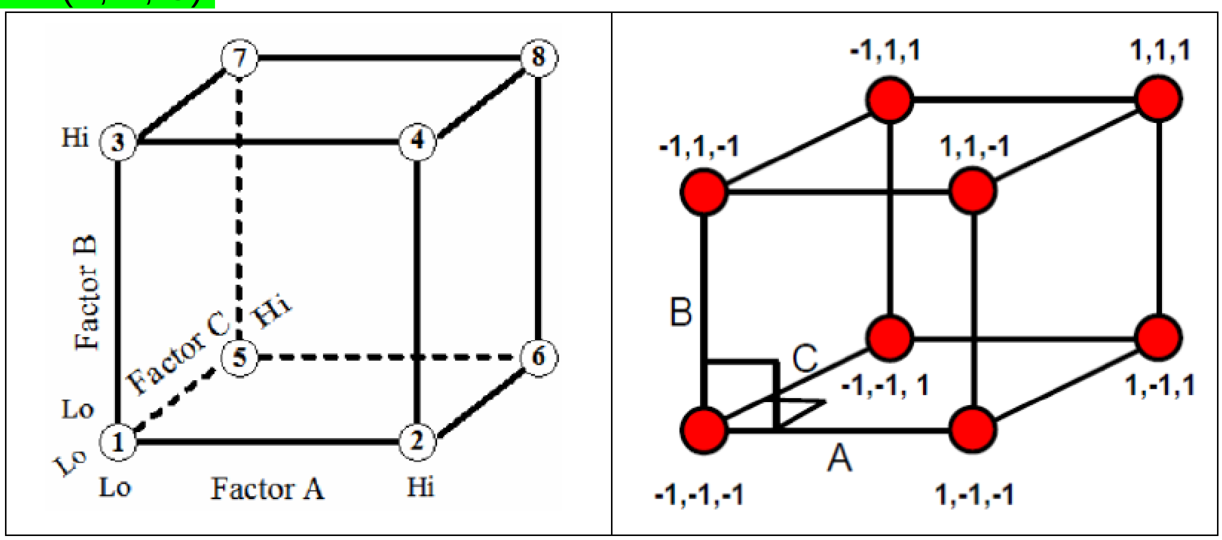


Fig. 1: Schematic diagram of 2^3 factorial (3 factors are investigated at 2 Levels) design

Definitions

- Factor – experimental variables that can be changed independently of each other. A variable is under the control of the experimenter. Factors are explanatory variables. A factor has 2 or more levels.
- Independent Variables: same as factors
- Continuous Variables: independent variables that can be changed continuously
- Response - the measured value of the result(s) from experiments
- Experimental unit - The unit to which the treatment is applied

- Main Effect: The effect of one independent factor (variable), ignoring the effects of all other independent variables. It is an effect is the difference in the means between the high and the low levels of a factor. The effect of a factor is defined to be the average change in the response associated with a change in the level of the factor. This is usually called a main effect.

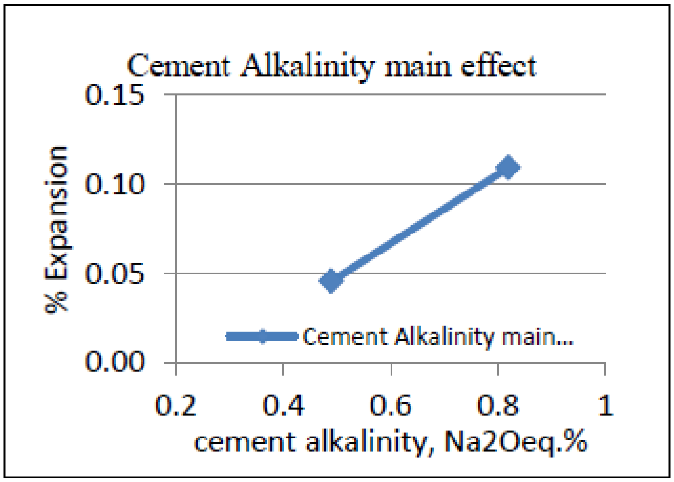


Fig. 2: Cement alkalinity main effect on expansion of Alkali Silica Reactive concrete specimen

- Interaction Effect: An interaction occurs when one factor effects the results differently depending on a second factor If the average change in response across the levels of one factor are not the same at all levels of the other factor, then we say there is an interaction between the factors.

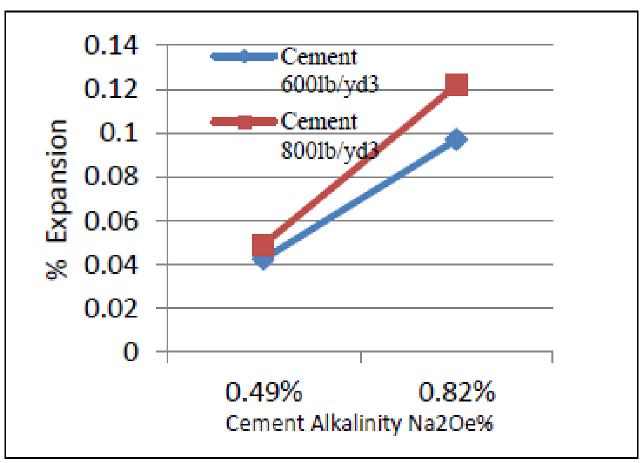


Fig. 3: Interaction between amount of Cement and Cement alkalinity on expansion of Alkali Silica Reactive concrete specimen

"There are two ways of defining an interaction between two factors A and B:

– If the average change in response between the levels of factor A is not the same at all levels of factor B, then an interaction exists between factors A and B.
– The lack of additivity of factors A and B, or the nonparallelism of the mean profiles of A and B, is called the interaction of A and B.
When we assume there is no interaction between A and B, we say the effects are additive.
Interpretation of the interaction plot: – Parallel lines usually indicate no significant interaction. – Severe lack of parallelism usually indicates a significant interaction. – Moderate lack of parallelism suggests a possible significant interaction may exist. • Statistical significance of an interaction effect depends on the magnitude of the MSE: For small values of the MSE(Mean Square Error), even small interaction effects (less nonparallelism) may be significant. • When an A ∗ B interaction is large, the corresponding main effects A and B may have little practical meaning. Knowledge of the A ∗ B interaction is often more useful than knowledge of the main effect. • We usually say that a significant interaction can mask the interpretation of significant main effects. That is, the experimenter must examine the levels of one factor, say A, at fixed levels of the other factor to draw conclusions about the main effect of A. • It is possible to have a significant interaction between two factors, while the main effects for both factors are not significant. This would happen when the interaction plot shows interactions in different directions that balance out over one or both factors (such as an X pattern). This type of interaction, however, is uncommon.” (http://www.math.montana.edu/jobo/st541/sec4a.pdf)

- Residual: the difference between the calculated and the experimental result

Each independent variable is a factor in the design.

Factorial experiments can be designed with one, two, three and more factors. Experiments with only one factor are often called simple comparative experiments. In these cases, t–test or ANOVA were used for analysis.
The simplest factorial design involves two variables which are named as factors, each at two levels.

Full factorial design
Full factorial design creates experimental points using all the possible combinations of the levels of the factors in each complete trial or replication of the experiments. The experimental design points in a full factorial design are the vertices of a hyper cube in the n-dimensional design space defined by the minimum and the maximum values of each of the factors. These experimental points are also called factorial points.
[Prasanta Sahoo, Tapan Kr. Barman, in Mechatronics and Manufacturing Engineering, 2012]

The notation used to denote factorial experiments conveys a lot of information. When a design is denoted a 2^3 factorial, this identifies the number of factors (3); how many levels each factor has (2); and how many experimental conditions there are in the design (2^3=8). Similarly, a 2^5 design has five factors, each with two levels, and 2^5=32 experimental conditions; and a 3^2 design has two factors, each with three levels, and 3^2=9 experimental conditions. Factorial experiments can involve factors with different numbers of levels. A $2^4 3$ design has five factors—four with two levels and one with three levels—and has 16×3=48 experimental conditions.

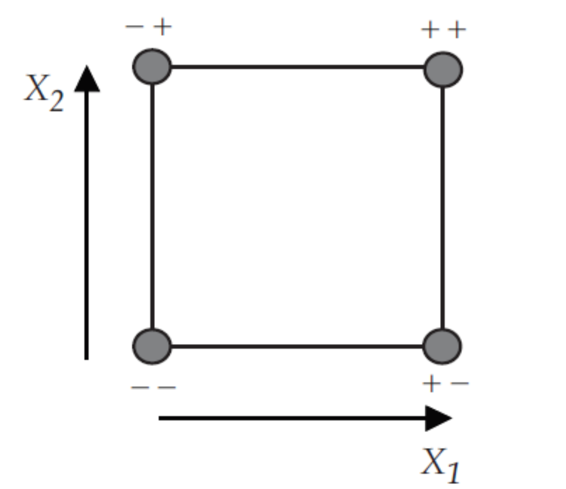


Fig.4 : The experiment in a design with two variables with two levels

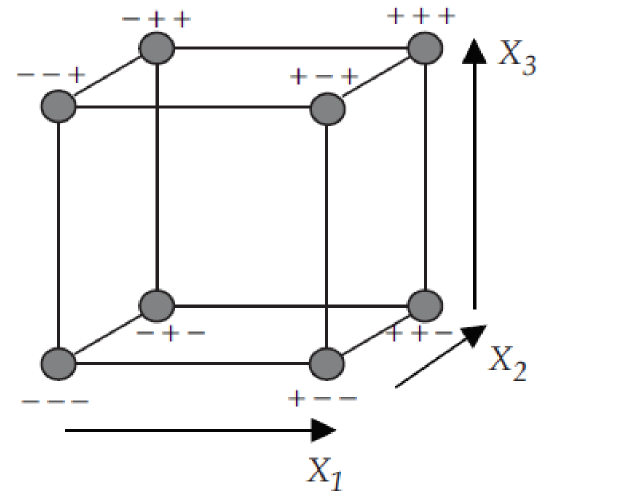


Fig. 5 : The experiment in a design with three variables with two levels

Courtesy: General Introduction to Design of Experiments (DOE)
https://www.researchgate.net/publication/221913304

Factorial Design: Historical perspective (DOI: 10.21533/pen.v5i3.145- Benjamin Durakovic - International University of Sarajevo)

One Factor at a Time (OFAT) was very popular scientific method dominated until early nineteen century. In this method one variable/factor is tested at a time while the other variables are constrained except the investigated one. Testing multiple variables at a time is better especially in cases where data must be analyzed carefully. In the 1920s and 1930s Ronald A. Fisher conducted a research in agriculture with the aim of increasing yield of crop in the UK. Getting data and was challenging e.g. if he relayed on his traditional method ANOVA (F-test, means Fisher - test) he may plant a crop in spring and get results in fall which is too long for getting data. Finally, he came up with design of experiment and officially he was the first one who started using DOE. In 1935, he wrote a book on DOE, in which he explained how valid conclusion could be drawn from the experiment in presence of nuisance factors. He analyzed presence of nuisance factors with fluctuation of weather conditions (temperature, rainfall, soil condition). Credit for Response Surface Method (RSM) belongs to George Box who is also from the UK. He was concerned with experimental design procedures for process optimization.

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Pareto Analysis: A tool for deciding what factors (variables) to work on.

History of Pareto Analysis:

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- The Pareto effect (written in 1896) is named after Vilfredo Pareto, an Italian economist and sociologist who lived from 1848 to 1923.
- ‘‘in any series of elements to be controlled, a selected small factor in terms of the number of elements almost always accounts for a large factor in terms of effort’’.

Fig. 6: Vilfredo Pareto

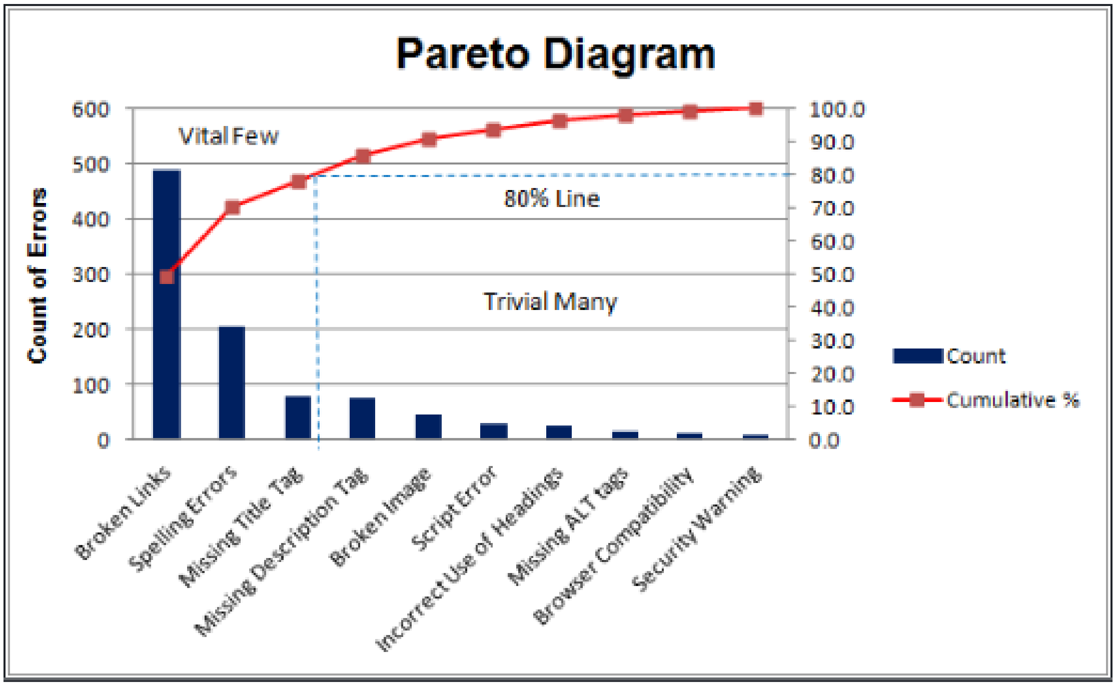

Fig. 7: Pareto Diagram for Website Errors

Pareto Analysis

Website Errors

Error (Cause)	Count	Cumulative Count	Cumulative %
Broken Links	349	349	43.7
Spelling Errors	169	518	64.8
Missing Title Tag	79	597	74.7
Missing Description Tag	77	674	84.4
Broken Image	45	719	90.0
Script Error	30	749	93.7
Incorrect Use of Headings	15	764	95.6
Missing ALT tags	14	778	97.4
Browser Compatibility	12	790	98.9
Security Warning	9	799	100.0