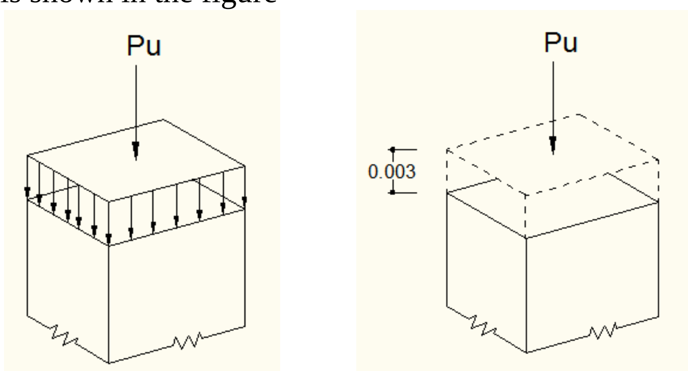
Column: Part 2
Courtesy of Dr. Latifee's IMI research group, Text books and others

Design for Axial Load:

With negligible moment and considerable amount of concentric axial load, failure occurs when the concrete strain reaches ultimate stain of 0.003. However, considering the design safety factors, the ultimate strain may be restricted to 0.002. The stress and strain distribution in pure axially loaded column is shown in the figure



(a) (b)

Figure: (a) stress distribution in axially loaded column, (b) maximum strain at compression failure

Ideally, if a column is subjected the pure axial load, concrete and reinforcing steel undergo same amount of shortening. Concrete reaches its maximum strength at $0.85f'_c$ (ksi or MPa) first. Then, concrete continues to yield until steel reaches its yield strength, f_y (ksi or MPa). The nominal strength of a reinforced concrete column is the sum of contribution from concrete [$0.85f'_c\,(A_g-A_{st})$, where f'_c is compressive strength of concreter, A_g is gross area of column; A_{st} is areas of reinforcing steel] and reinforcing longitudinal steel [$A_{st}f_y$]. Thus the nominal strength of a column is,

$$P_n = 0.85f'_c\,(A_g-A_{st})+A_{st}f_y$$

$$\Rightarrow\ P_n = 0.85f'_cA_g+A_{st}\,(f_y-0.85f'_c)$$

$$\Rightarrow\ P_n = A_g\,[0.85f'_c+\rho_g(f_y-0.85\,f'_c)]$$

For design purpose, ACI 10.3.6 specify column strength as follows:

For tied column,

$\varphi P_{n(max)} = 0.80\varphi A_g\,[0.85f'_c+\rho_g(f_y-0.85\,f'_c)]$ (kips) ACI Eq. (10-2)

For spiral column,

$\varphi P_{n(max)} = 0.85\varphi A_g\,[0.85f'_c+\rho_g(f_y-0.85\,f'_c)]$ (kips) ACI Eq. (10-1)

Where A_g= gross area of column, in^2

A_{st}= total area of longitudinal reinforcement, in^2

$\rho_g = A_{st}/A_g$, φ= strength reduction factor

The factors 0.85φ and 0.8φ are considering the effect of confinement of column ties, strength reduction due to failure mode and accidental eccentricities. **The strength reduction factor φ is taken as 0.75 for columns with spiral reinforcement and 0.65 for with tie reinforcement.**

Preliminary column sizing: To minimize the design time, effective initial selection of column size is very important. Generally, a preliminary column size should be determined using a low percentage of reinforcement to facilitate the possibility to provide required additional reinforcement in the final design without changing the column size. **1% to 2% reinforcement ratios usually results in the most economical design. Though the maximum amount of steel could be 8% (ACI 10.9), most columns are designed with ratios below 4 % of the gross cross-sectional area of the column considering the economy and to avoid congestion of the reinforcement**. Concrete is more cost effective than reinforcement for carrying compressive axial load; thus, it is more economical to use larger column sizes with lesser amounts of reinforcement. **Also, columns with a smaller number of larger bars are usually more economical than columns with a larger number of smaller bars.**

The design charts presented in figures below are based on ACI Eq. (10-2) can be used predict the size required for non-slender tied square column for design factored load, P_u. For different combination of concrete strength and steel grade with reinforcement ratios between 1% and 8% are considered. These can be used for column sizes from 10 in. to 24 in. with negligible eccentricity (no more than 0.1 h, here h is the size of column). The figures below can also be used to determine the required column area for corresponding axial load. To fast-track the selection procedure with minimum reinforcement ratio (1%), table 1 can be used directly.

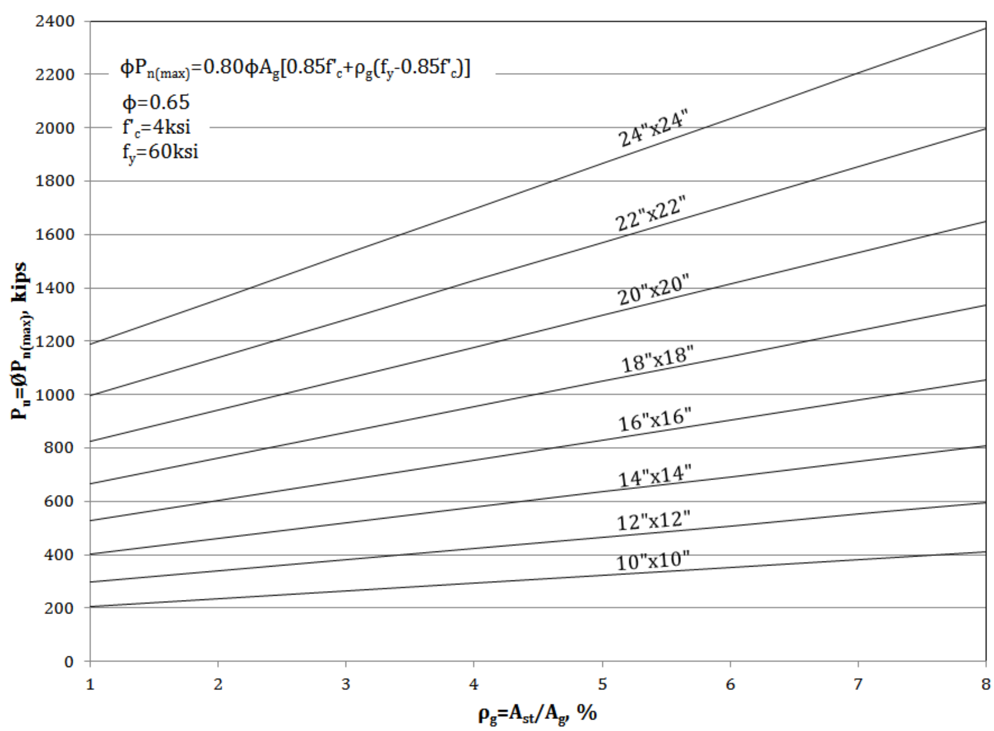


Figure: Design chart for non-slender square tied column (f'_c= 4 ksi, f_y= 60 ksi)

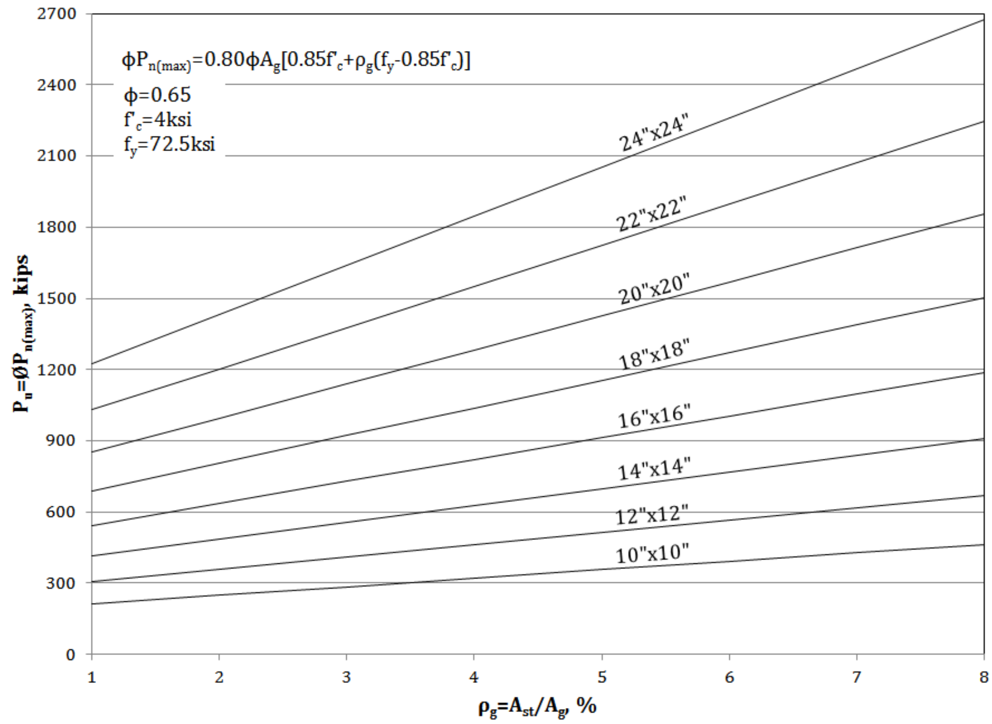


Figure: Design chart for non-slender square tied column (f'_c= 4 ksi, f_y= 72.5 ksi)

Table 1: Design table for non-slender tied column with minimum steel ration (1%)

Steel grade, f_y (ksi)	**Concrete strength, f'_c (ksi)**	**Minimum steel ratio, $\rho_{g(min)}$**	**Required column area, A_g (in^2)**
	12		**$P_u/5.56$**
	11		**$P_u/5.12$**
	10		**$P_u/4.68$**
	9		**$P_u/4.25$**
	8		**$P_u/3.81$**
60	**7**	**1%**	**$P_u/3.37$**
	6		**$P_u/2.93$**
	5		**$P_u/2.49$**
	4		**$P_u/2.06$**
	3.5		**$P_u/1.84$**
	3		**$P_u/1.62$**
	12		**$P_u/5.62$**
	11		**$P_u/5.19$**
	10		**$P_u/4.75$**
	9		**$P_u/4.31$**
	8		**$P_u/3.87$**
72.5	**7**	**1%**	**$P_u/3.44$**
	6		**$P_u/3.00$**
	5		**$P_u/2.56$**
	4		**$P_u/2.12$**
	3.5		**$P_u/1.90$**
	3		**$P_u/1.68$**

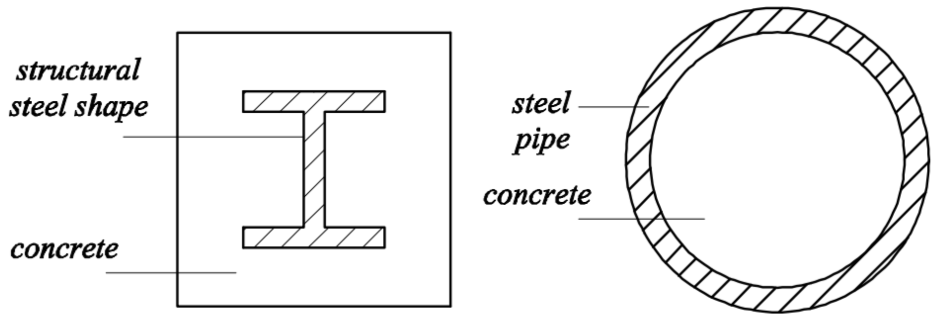


Figure: Composite Column

Behavior of Tied and Spirally-Reinforced Columns
Axial loading tests have proven that tied and spirally reinforced columns having the same cross sectional areas of concrete and steel reinforcement behave in the same manner up to the ultimate load, as shown in Figure below. At that load tied columns fail suddenly due to excessive cracking in the concrete section followed by buckling of the longitudinal reinforcement between ties within the failure region. For spirally reinforced columns, once the ultimate load is reached, the concrete shell covering the spiral starts to peel off. Only then, the spiral comes to action by providing a confining force to the concrete core, thus enabling the column to sustain large deformations before final collapse occurs.

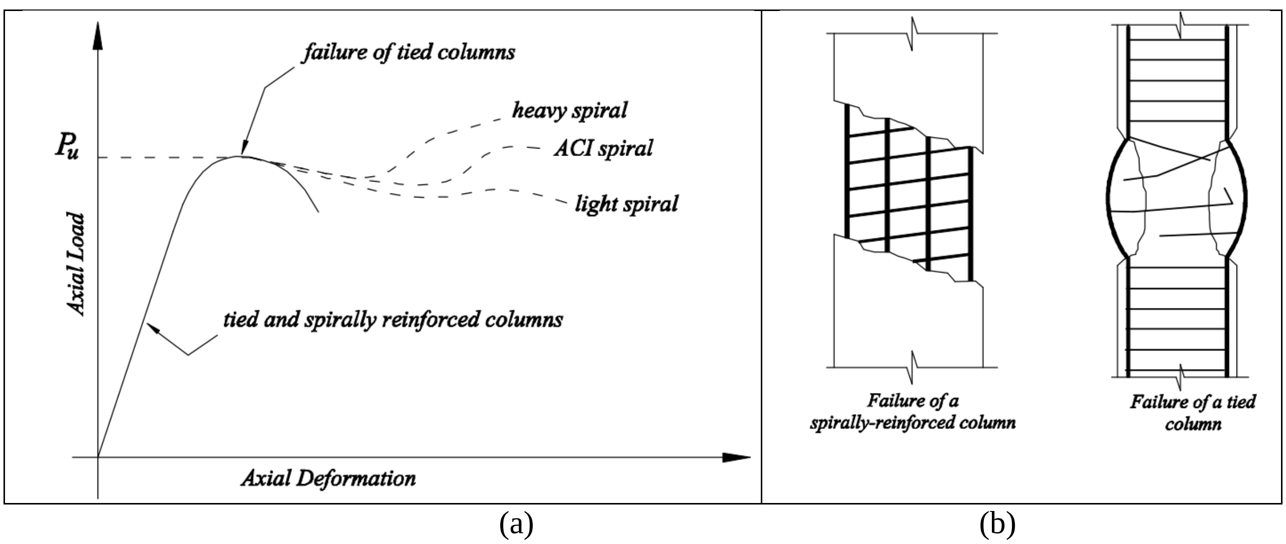


Figure: Failure of columns; (a) behavior of tied and spirally reinforced columns; (b) failure of columns

Factored Loads
Load factors for dead, live, wind or earthquake live loads combinations are shown in the Table below.

Table: Load factors for dead, live, wind or earthquake live loads combinations

Loads	Required Strength
Dead (D) and Live (L)	$1.4D$ $1.2\ D+1.6\ L$
Dead (D), Live (L) and wind (W)	$1.2\ D+1.0\ L$ $1.2\ D+0.8\ W$ $1.2\ D+1.6\ W+1.0\ L$ $0.9\ D+1.6\ W$
Dead (D), Live (L) and Earthquake (E)	$1.2\ D+1.0\ L+1.0\ E$ $0.9\ D+1.0\ E$

Write Short note on Ties in the column with detail figures

Ties in the column

When tied columns are used, the ties shall not be less than #3, provided that the longitudinal bars are #10 or smaller.

The minimum size is #4 for longitudinal bars larger than #10 and for bundled bars. Deformed wire or welded wire fabric with an equivalent area may also be used (ACI 7.10.5.1).

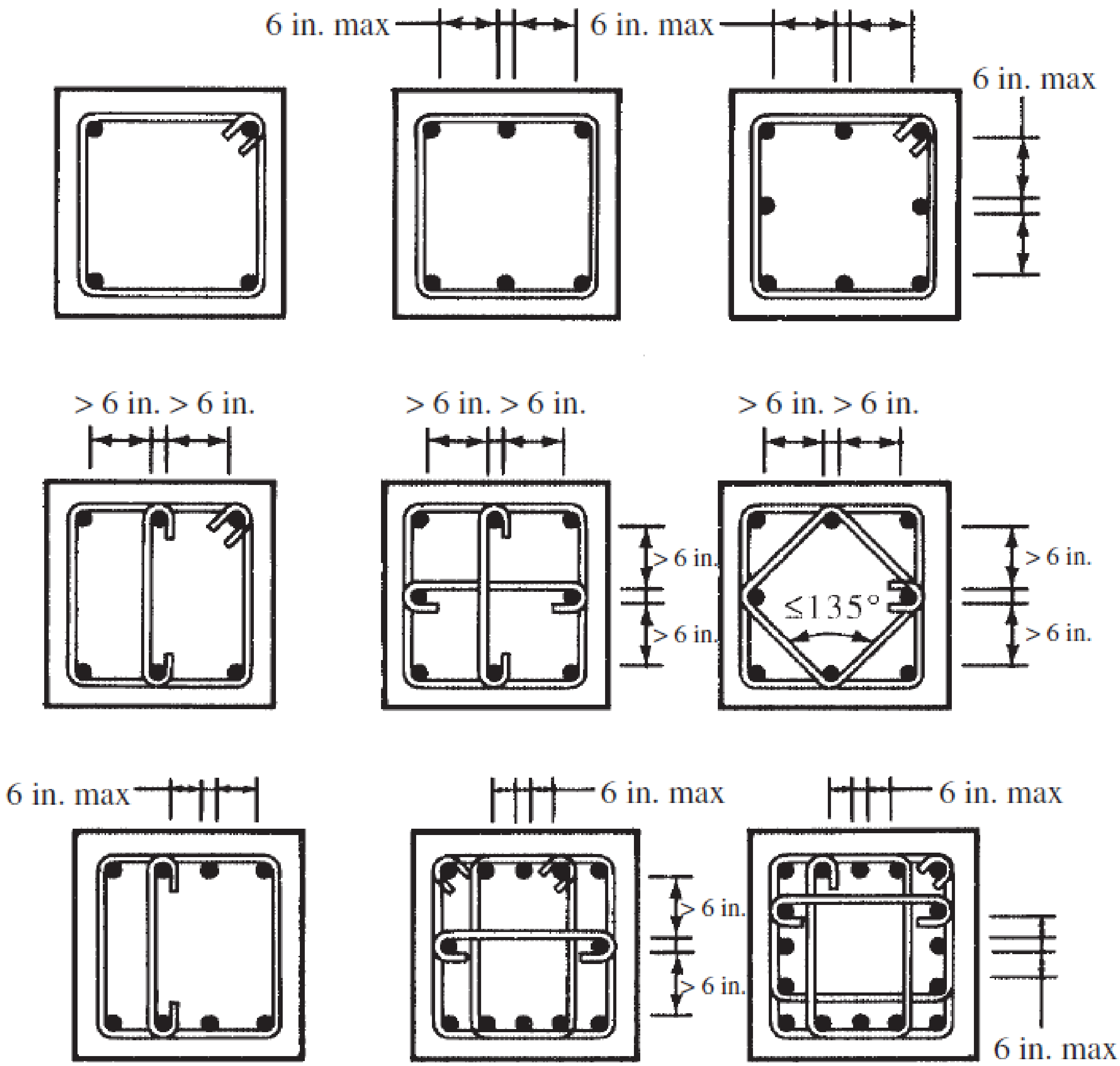
In SI units, ties should not be less than #10 for longitudinal bars #32 or smaller and #13 for larger longitudinal bars.

Spacing of Tie bar:

ACI Code 7.10.5.2 specifies that vertical spacing of ties is not to exceed the smallest of:

- **16 times longitudinal bar diameter.**
- **48 times tie diameter.**
- **Least cross sectional dimension.**

ACI Code 7.10.5.3 specifies that ties are arranged in such a way that every corner and alternate longitudinal bar is to have lateral support provided by the corner of a tie with an included angle of not more than 135 degrees. Besides, no longitudinal bar is to be farther than 15 cm clear on each side along the tie from such a laterally supported bar. When longitudinal bars are located around the perimeter of a circle, circular ties are used.


6 in. max
6 in. max
6 in. max
> 6 in. > 6 in.
> 6 in. > 6 in.
> 6 in. > 6 in.
> 6 in.
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≤135°
> 6 in.
> 6 in.
6 in. max
6 in. max
6 in. max
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> 6 in.
6 in. max

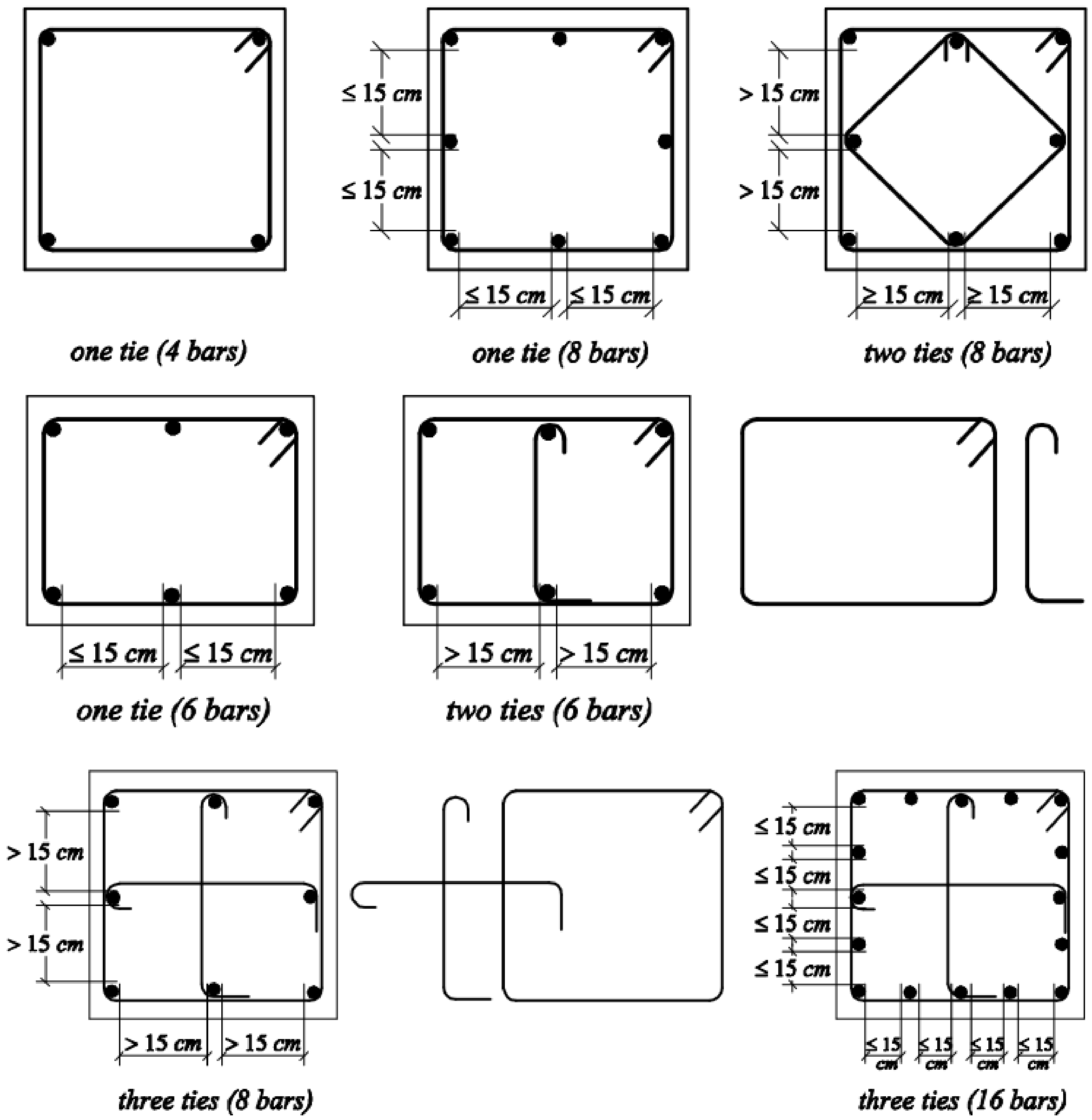


Figure: Tie arrangements

	Minimum cover, mm
Beams, Columns :	
Primary reinforcement, Ties, stirrups, spirals	40

Example 1:

Design a square tied column to support an axial dead load D of 130 k and an axial live load L of 180 k. Initially assume that 2% longitudinal steel is desired, $f'_c = 4000$ psi, and $f_y = 60{,}000$ psi.

SOLUTION

$$P_u = (1.2)(130 \text{ k}) + (1.6)(180 \text{ k}) = 444 \text{ k}$$

Selecting Column Dimensions

$$\phi P_n = \phi 0.80[0.85 f'_c (A_g - A_{st}) + f_y A_{st}] \qquad \text{(ACI Equation 10-2)}$$

$$444 = (0.65)(0.80)[(0.85)(4 \text{ ksi})(A_g - 0.02A_g) + (60 \text{ ksi})(0.02A_g)]$$

$$A_g = 188.40 \text{ in.}^2 \qquad \underline{\underline{\text{Use 14 in.} \times \text{14 in.} \; (A_g = 196 \text{ in.}^2)}}$$

Selecting Longitudinal Bars

Substituting into column equation with known A_g and solving for A_{st}, we obtain from ACI Equation 10-2,

$$444 = (0.65)(0.80)[(0.85)(4 \text{ ksi})(196 \text{ in.}^2 - A_{st}) + (60 \text{ ksi})A_{st}]$$

$$A_{st} = 3.31 \text{ in.}^2 \qquad \underline{\underline{\text{Use 6 \#7 bars } (3.61 \text{ in.}^2)}}$$

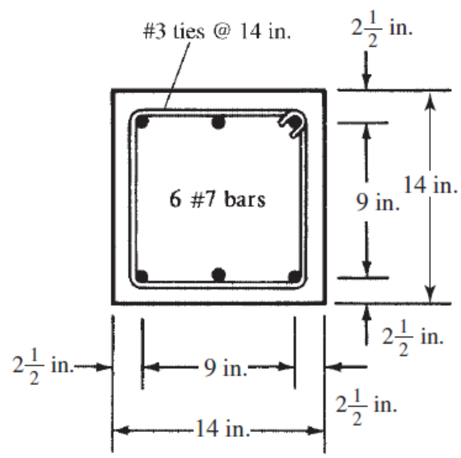
Design of Ties (Assuming #3 Bars)

Spacing:

(a) 48 in. $\times \frac{3}{8}$ in. $=$ 18 in.

(b) 16 in. $\times \frac{7}{8}$ in. $=$ 14 in. ←

(c) Least dim. $=$ 14 in. ← Use #3 ties @ 14 in.



Check Code Requirements

Following are the ACI Code limitations for columns. Space is not taken in future examples to show all of these essential checks, but they must be made.

(7.6.1) Longitudinal bar clear spacing $= \frac{9}{2}$ in. $- \frac{7}{8}$ in. $= 3.625$ in. > 1 in. and d_b of $\frac{7}{8}$ in. OK

(10.9.1) Steel percentage $0.01 < \rho = \dfrac{3.61}{(14 \text{ in.})(14 \text{ in.})} = 0.0184 < 0.08$ OK

(10.9.2) Number of bars $= 6 >$ min. no. of 4 OK

(7.10.5.1) Minimum tie size $=$ #3 for #7 bars OK

(7.10.5.2) Spacing of ties OK

(7.10.5.3) Arrangement of ties OK

Example 2

Design an axially loaded short square tied column for $P_u = 2600$ kN if $f'_c = 28$ MPa and $f_y = 350$ MPa. Initially assume $\rho = 0.02$.

SOLUTION

Selecting Column Dimensions

$$\phi P_n = \phi 0.80[0.85 f'_c (A_g - A_{st}) + f_y A_{st}] \qquad \text{(ACI Equation 10-2)}$$

$$2600 \text{ kN} = (0.65)(0.80)[(0.85)(28 \text{ MPa})(A_g - 0.02A_g) + (350 \text{ MPa})(0.02A_g)]$$

$$A_g = 164\ 886 \text{ mm}^2$$

Use 400 mm × 400 mm ($A_g = 160\ 000$ mm^2)

Selecting Longitudinal Bars

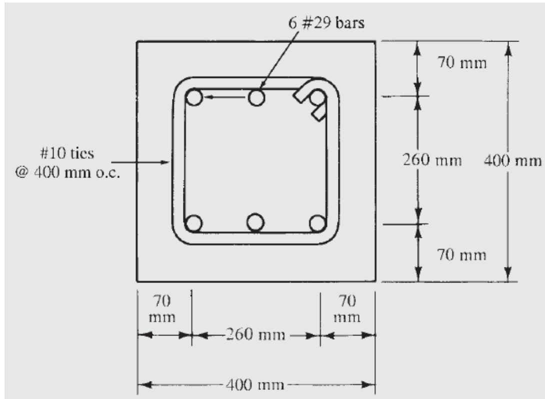
$$2600 \text{ kN} = (0.65)(0.80)[(0.85)(28 \text{ MPa})(160\ 000 \text{ mm}^2 - A_{st}) + (350 \text{ MPa})A_{st}]$$

$$A_{st} = 3654 \text{ mm}^2$$

Use 6 #29 (3870 mm^2)

Design of Ties (Assuming #10 SI Ties)

(a) 16 mm × 28.7 mm = 459.2 mm
(b) 48 mm × 9.5 mm = 456 mm
(c) Least col. dim. = 400 mm ← Use #10 ties @ 400 mm



Axial Load and Bending

Columns will bend under the action of moments, and those moments will tend to produce compression on one side of the columns and tension on the other. Depending on the relative magnitudes of the moments and axial loads, there are several ways in which the sections might fail. The column is assumed to reach its ultimate capacity when the compressive concrete strain reaches 0.003.

(a) *Large axial load with negligible moment*—For this situation, failure will occur by the crushing of the concrete, with all reinforcing bars in the column having reached their yield stress in compression.

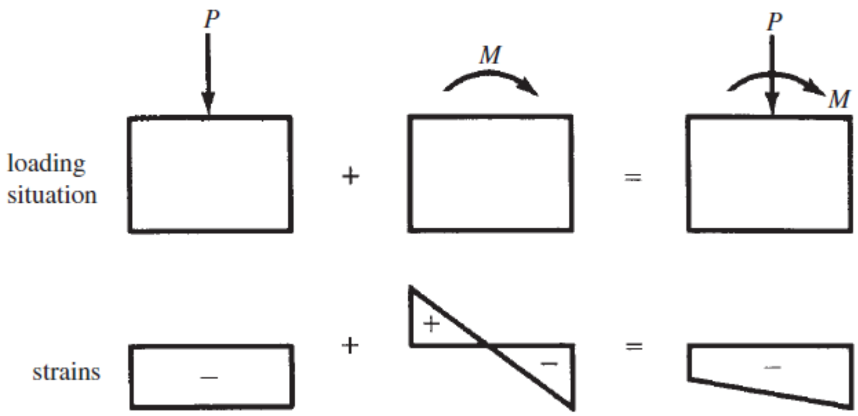
(b) *Large axial load and small moment such that the entire cross section is in compression*—When a column is subject to a small bending moment (i.e., when the eccentricity is small), the entire column will be in compression, but the compression will be higher on one side than on the other. The maximum compressive stress in the column will be $0.85f'_c$, and failure will occur by the crushing of the concrete with all the bars in compression.

(c) *Eccentricity larger than in case (b) such that tension begins to develop on one side of the column*—If the eccentricity is increased somewhat from the preceding case, tension will begin to develop on one side of the column, and the steel on that side will be in tension but less than the yield stress. On the other side, the steel will be in compression. Failure will occur as a result of the crushing of the concrete on the compression side.

(d) *A balanced loading condition*—As we continue to increase the eccentricity, a condition will be reached in which the reinforcing bars on the tension side will reach their yield stress at the same time that the concrete on the opposite side reaches its maximum compression, $0.85f'_c$. This situation is called the *balanced loading condition*.

(e) *Large moment with small axial load*—If the eccentricity is further increased, failure will be initiated by the yielding of the bars on the tensile side of the column prior to concrete crushing.

(f) *Large moment with no appreciable axial load*—For this condition, failure will occur as it does in a beam.

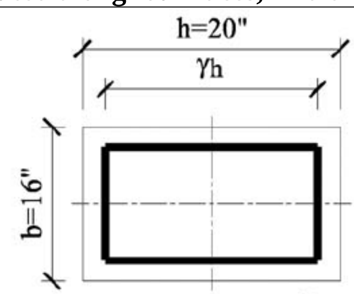


(a) Axial load (b) Moment (c) Axial load and moment

Example 3

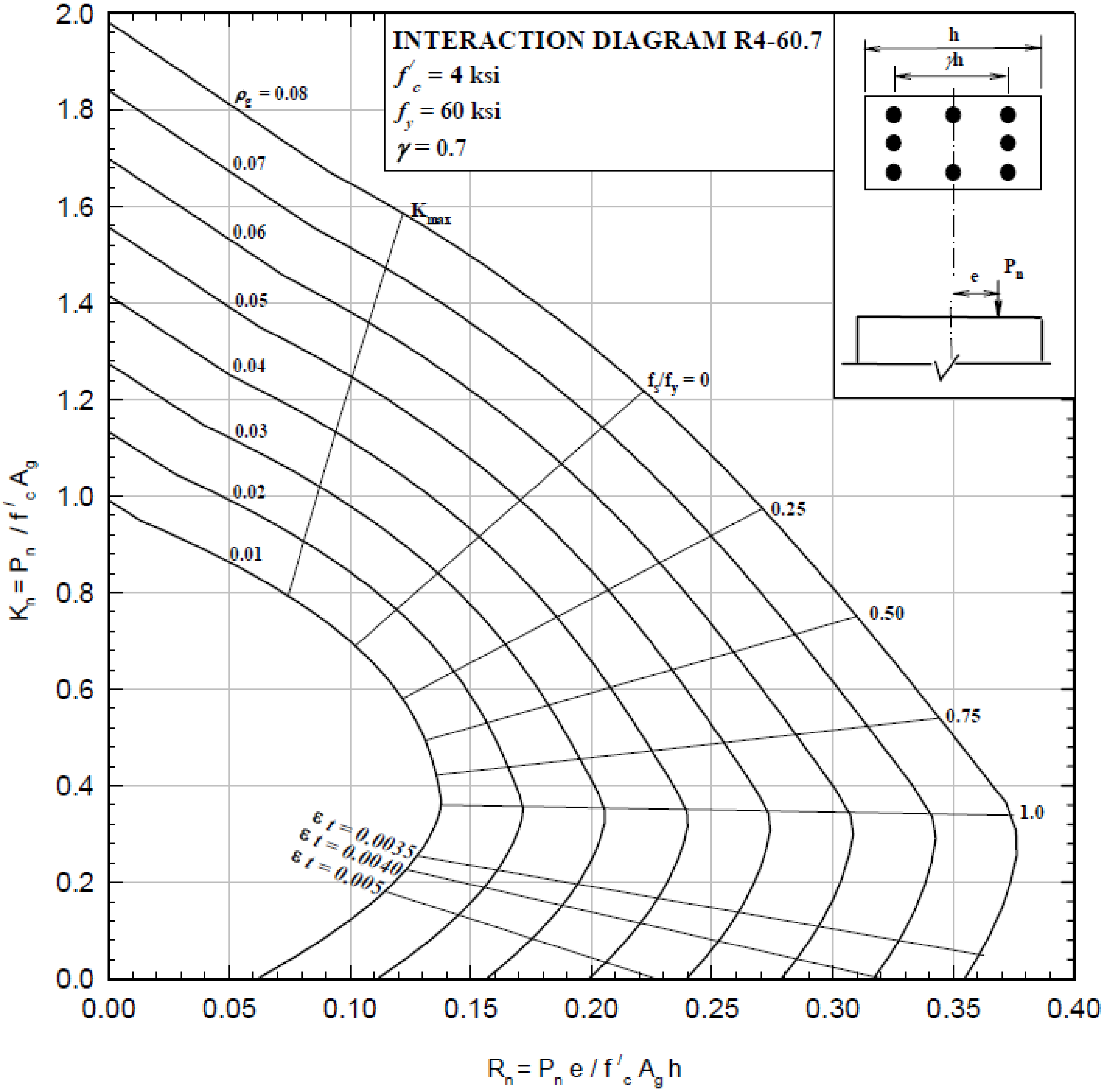
Required area of steel for a rectangular tied column with bars on four faces (slenderness ratio found to be below critical value)

For a rectangular tied column with bars equally distributed along four faces, find area of steel.

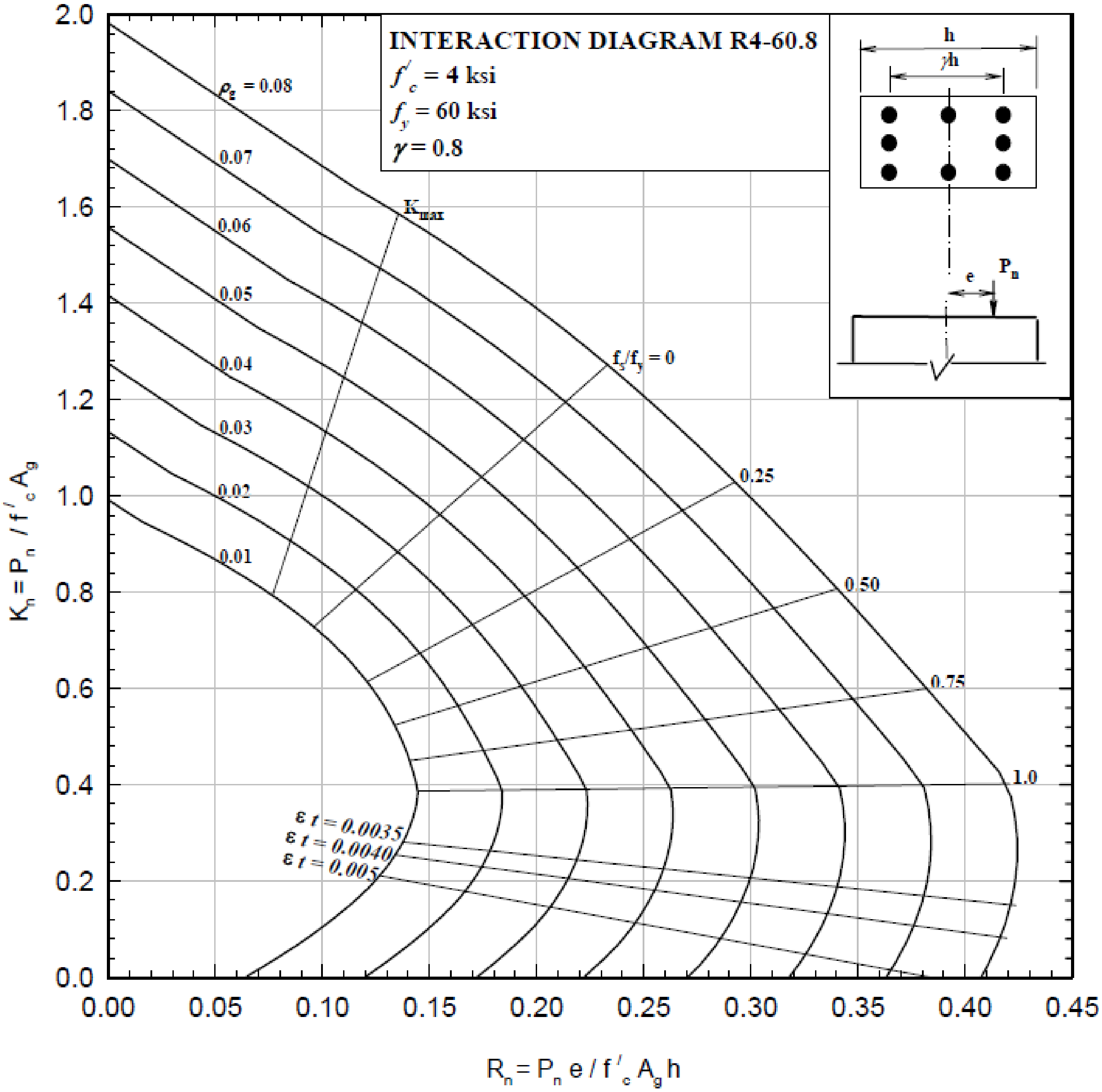
Materials Compressive strength of concrete $f_c^/ = 4$ ksi Yield strength of reinforcement $f_y = 60$ ksi Nominal maximum size of aggregate is 1 in.	

Procedure	Calculation	ACI 318-05 Section	Design Aid
Determine column section size.	Given: h = 20 in. b = 16 in.		
Determine reinforcement ration ρ_g using known values of variables on appropriate interaction diagram(s) and compute required cross section area A_{st} of longitudinal reinforcement. A) Compute $K_n = \frac{P_n}{f'_c A_g}$ B) Compute $R_n = \frac{M_n}{f'_c A_g h}$ C) Estimate $\gamma \approx \frac{h - 5}{h}$ D) Determine the appropriate interaction diagram(s) E) Read ρ_g for k_n and R_n values from appropriate interaction diagrams F) Compute required A_{st} from $A_{st}=\rho_g A_g$	P_n= 800 kip M_n = 5600 kip-in. h = 20 in. b = 16 in. A_g = b x h = 20 x 16 = 320 in.^{2} $K_n = \frac{800}{(4)(320)} = 0.625$ $R_n = \frac{5600}{(4)(320)(20)} = 0.22$ $\gamma \approx \frac{20 - 5}{20} = 0.75$ For a rectangular tied column with bars along four faces, $f'_c = 4$ ksi, $f_y = 60$ ksi, and an estimated γ of 0.75, use R4-60.7 and R4-60.8. For k_n= 0.625 and R_n= 0.22 Read $\rho_g = 0.041$ for $\gamma = 0.7$ and $\rho_g = 0.039$ for $\gamma = 0.8$ Interpolating; $\rho_g = 0.040$ for $\gamma = 0.75$ Required $A_{st} = 0.040 \times 320$ in.^{2} = 12.8 in^2	10.2 10.3	Columns 3.2.2 (R4-60.7) and 3.2.3 (R4-60.8)

COLUMNS 3.2.2 - Nominal load-moment strength interaction diagram, R4-60.7



COLUMNS 3.2.3 - Nominal load-moment strength interaction diagram, R4-60.8



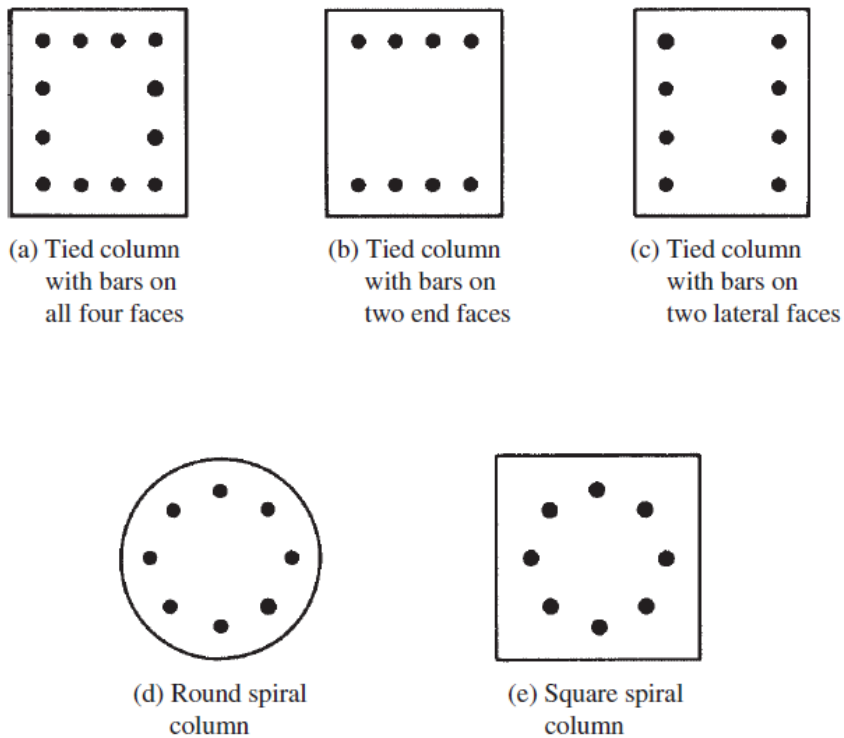


Figure: Column cross sections for normalized interaction curves

Example 4:

The short 14-in. × 20-in. tied column of Figure 10.17 is to be used to support the following loads and moments: $P_D = 125$ k, $P_L = 140$ k, $M_D = 75$ ft-k, and $M_L = 90$ ft-k. If $f'_c = 4000$ psi and $f_y = 60{,}000$ psi, select reinforcing bars to be placed in its end faces only using appropriate ACI column interaction diagrams.

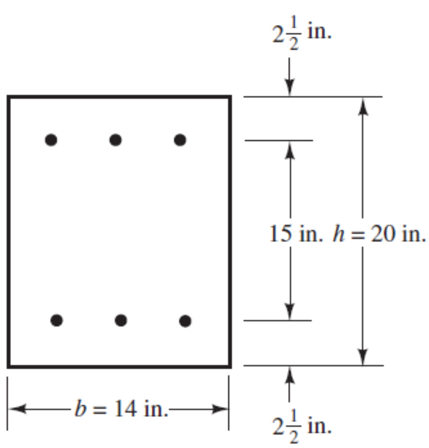


FIGURE 10.17 Column cross section

SOLUTION

$$P_u = (1.2)(125\text{ k}) + (1.6)(140\text{ k}) = 374\text{ k}$$

$$P_n = \frac{374\text{ k}}{0.65} = 575.4\text{ k}$$

$$M_u = (1.2)(75\text{ ft-k}) + (1.6)(90\text{ ft-k}) = 234\text{ ft-k}$$

$$M_n = \frac{234\text{ ft-k}}{0.65} = 360\text{ ft-k}$$

$$e = \frac{(12\text{ in/ft})(360\text{ ft-k})}{575.4\text{ k}} = 7.51\text{ in.}$$

$$\gamma = \frac{15\text{ in.}}{20\text{ in.}} = 0.75$$

Compute values of K_n and R_n

$$K_n = \frac{P_n}{f'_c A_g} = \frac{575.4\text{ k}}{(4\text{ ksi})(14\text{ in.} \times 20\text{ in.})} = 0.513$$

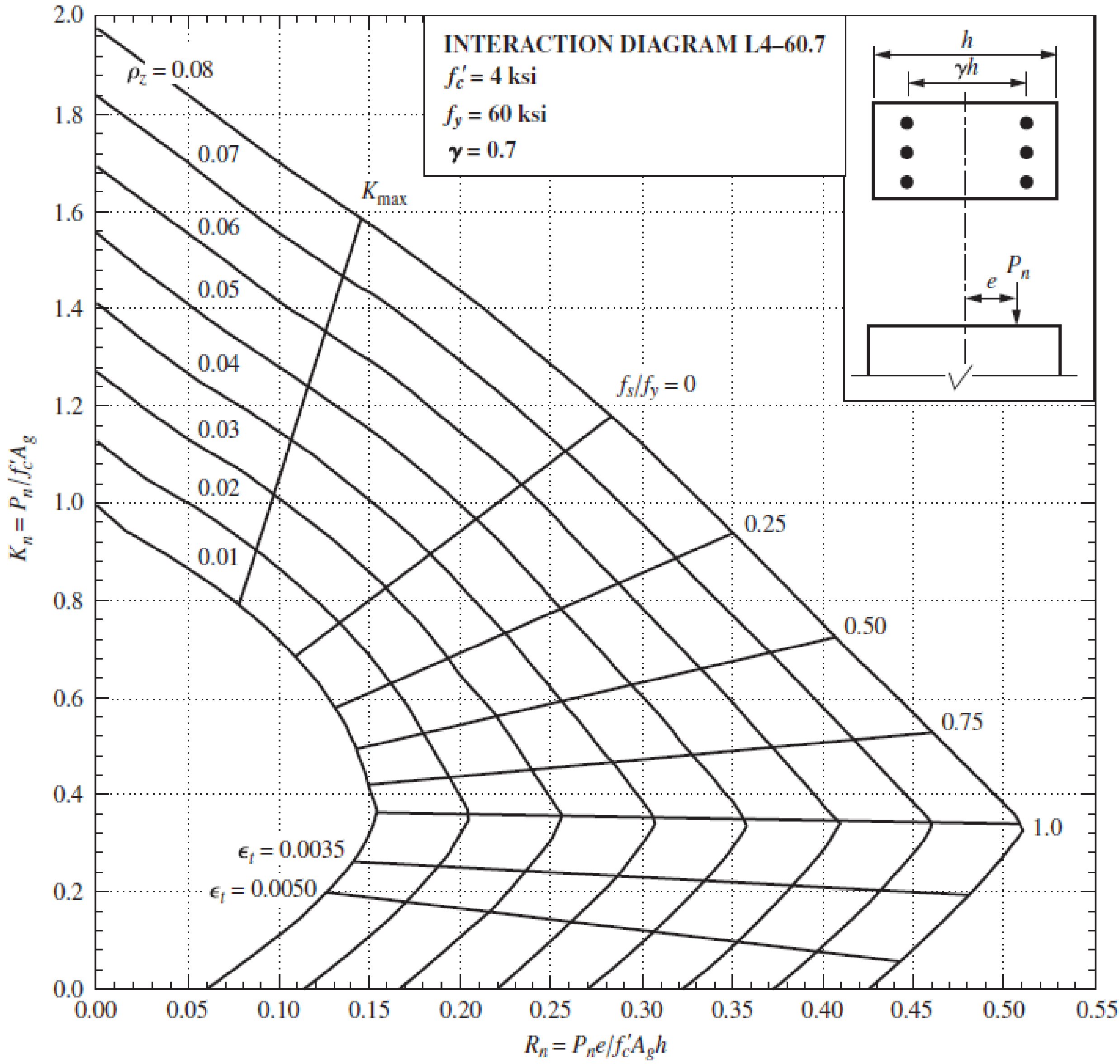
$$R_n = \frac{P_n e}{f'_c A_g h} = \frac{(575.4\text{ k})(7.51\text{ in.})}{(4\text{ ksi})(14\text{ in.} \times 20\text{ in.})(20\text{ in.})} = 0.193$$

The value of γ falls between γ values for Graphs 3 and 4 of Appendix A. Therefore, interpolating between the two as follows:

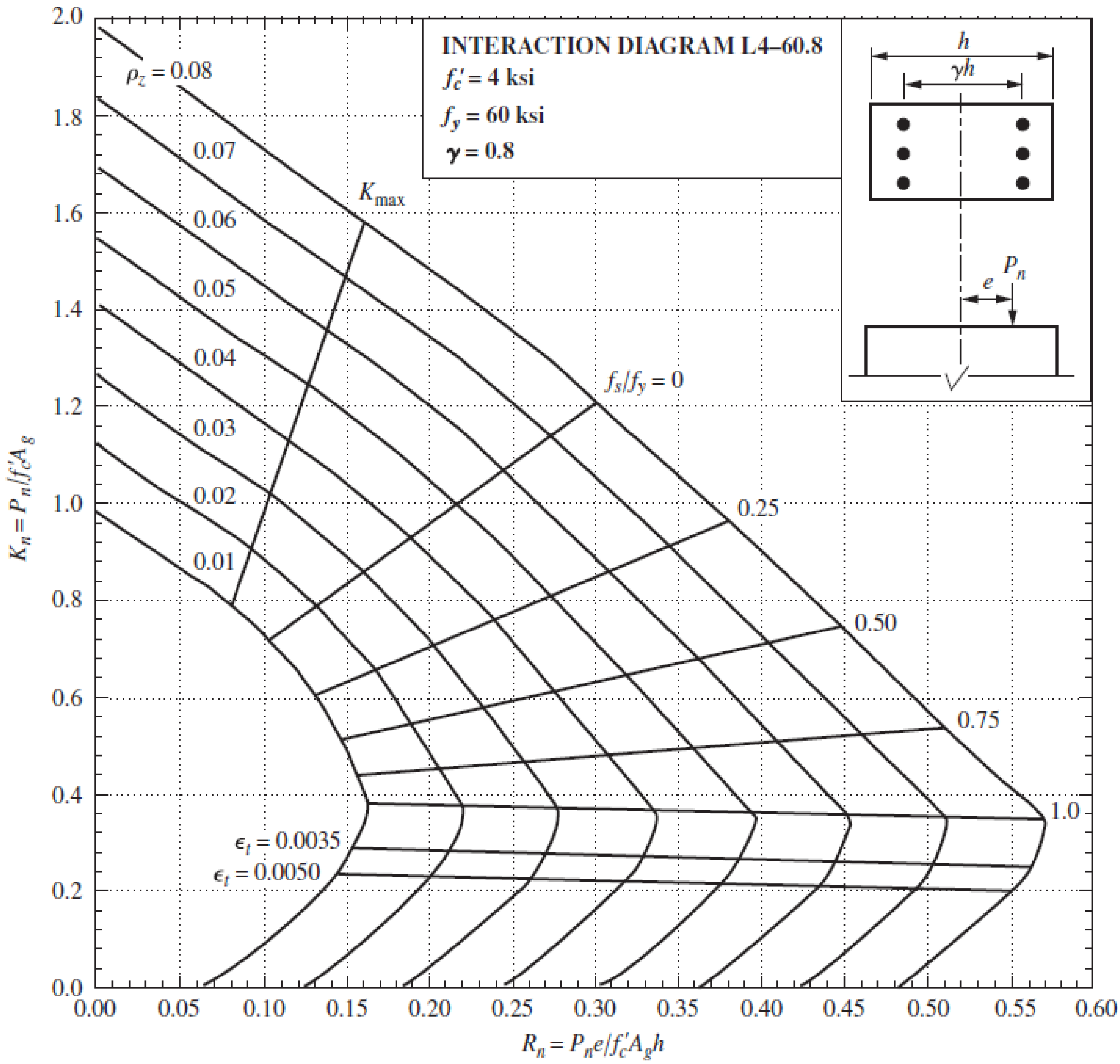
γ	0.70	0.75	0.80
ρ_g	0.0220	0.0202	0.0185

$$A_s = \rho_g bh = (0.0202)(14\text{ in.})(20\text{ in.}) = 5.66\text{ in.}^2$$

Use 6 #9 bars = 6.00 in.^2



GRAPH 3 Column interaction diagrams for rectangular tied columns with bars on end faces only. (Published with the permission of the American Concrete Institute.)



GRAPH 4 Column interaction diagrams for rectangular tied columns with bars on end faces only. (Published with the permission of the American Concrete Institute.)