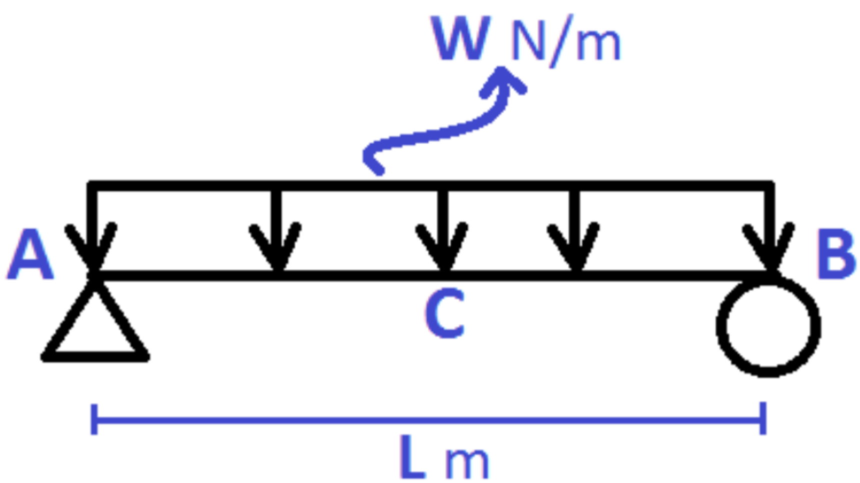
Center Point Deflection of a simply supported beam due to UDL-uniformly distributed loading

Q. Given the simply supported beam under a UDL (as shown in the figure below), determine Δ_{C}.



Solution: The equation we work with is $\Delta=\int \frac{M_0\, M_1}{EI}\, dx$,

Where $\mathbf{M_o}$ = bending moment distribution due to actual or real loading

$\mathbf{M_1}$ = bending moment distribution due to virtual or unit loading

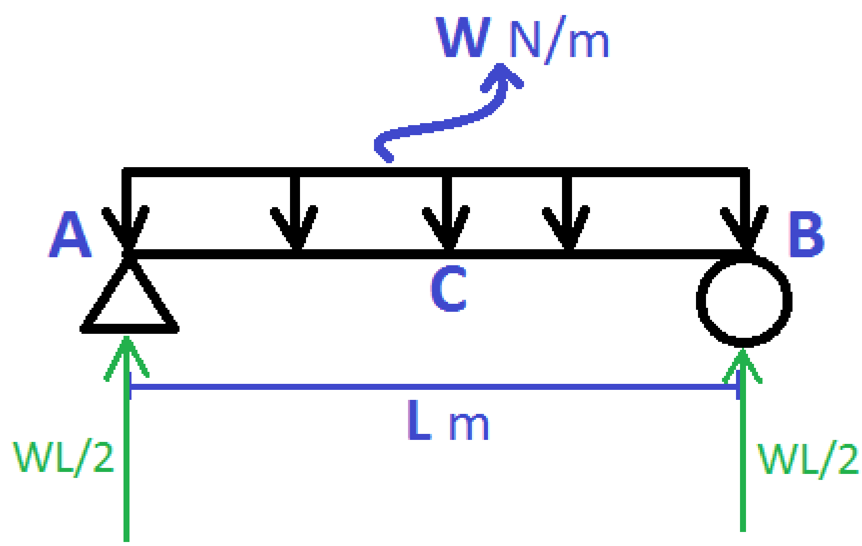
E = modulus of elasticity of the material of beam

I = moment of inertia of beam section

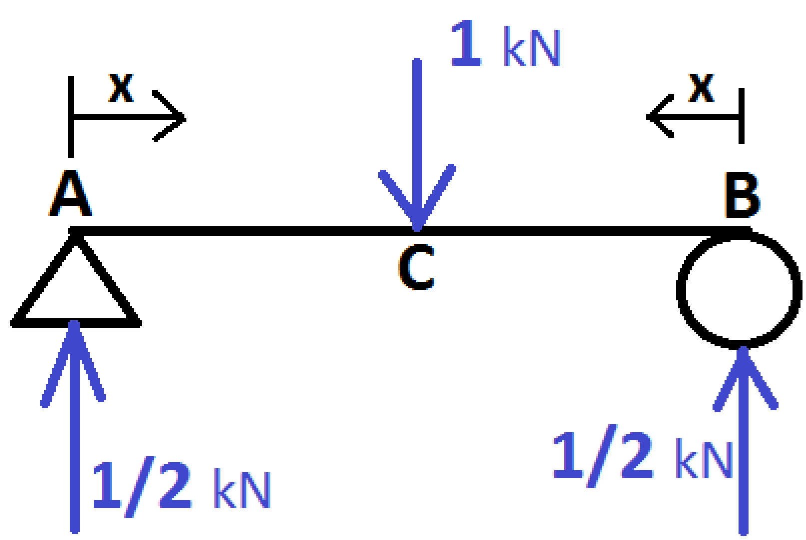
L = beam span

$\mathbf{\Delta_c}$ = deflection at point C

The diagram for M_0 is as follows-



The diagram for M_1 is as follows-



We know that $\Delta = \int \frac{M_0\,M_1}{EI}\,dx$

Hence,

$$EI\Delta_C = 2\int_0^{\frac{L}{2}} \mathbf{A{\rightarrow}C} \left\{\frac{wL}{2}.x - \frac{wx^2}{2}\right\}\left\{\frac{x}{2}\right\} dx$$

$$= 2\int_0^{\frac{L}{2}} \left\{\frac{wLx^2}{4} - \frac{wx^3}{4}\right\} dx$$

$$= 2\left[\frac{wL.x^3}{12} - \frac{wx^4}{16}\right]_0^{\frac{L}{2}}$$

$$= \frac{5}{384} wL^4$$

$$\therefore \Delta_C = \frac{5\,wL^4}{384\,EI}$$

Done by: Ms. Sama Ahmed

Checked by: Dr. Latifee, December 9, 2015