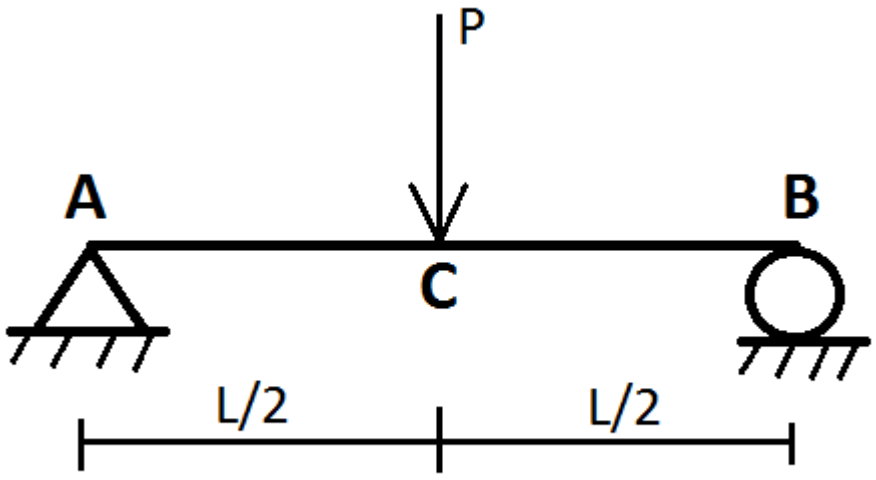
Center Point Deflection for simply supported beam due to single load by Unit Load/ Virtual Work Method

Question: **Find the deflection at C of the following simply-supported beam.**



Solution:

The Deflection, $\Delta_c = \int \frac{M_0 M_1}{EI} dx$,

Where $\mathbf{M_o}$ = Bending moment distribution due to actual or real loading

$\mathbf{M_1}$ = Bending moment distribution due to virtual or unit loading

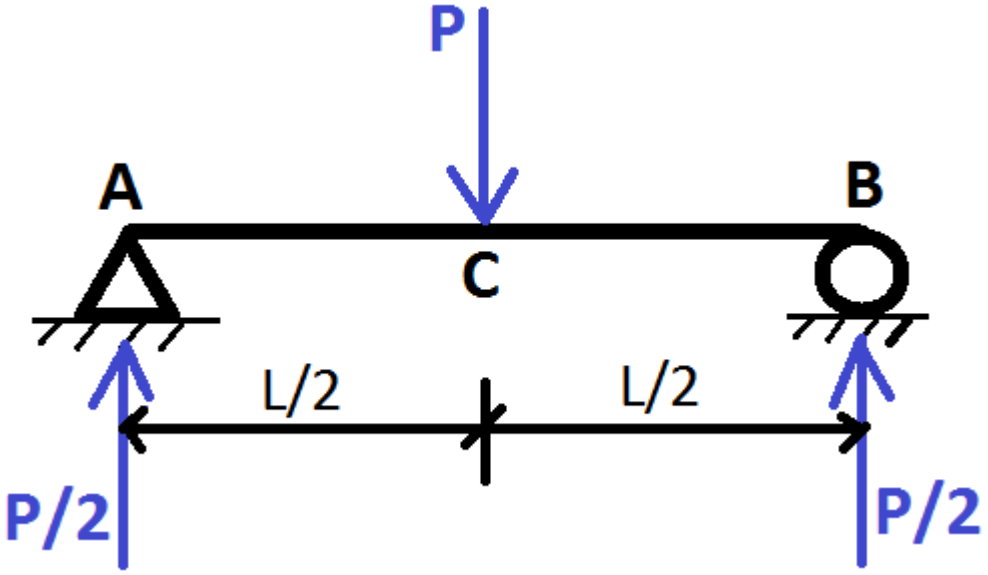
E = Modulus of elasticity of the material of beam

I = Moment of inertia of beam section

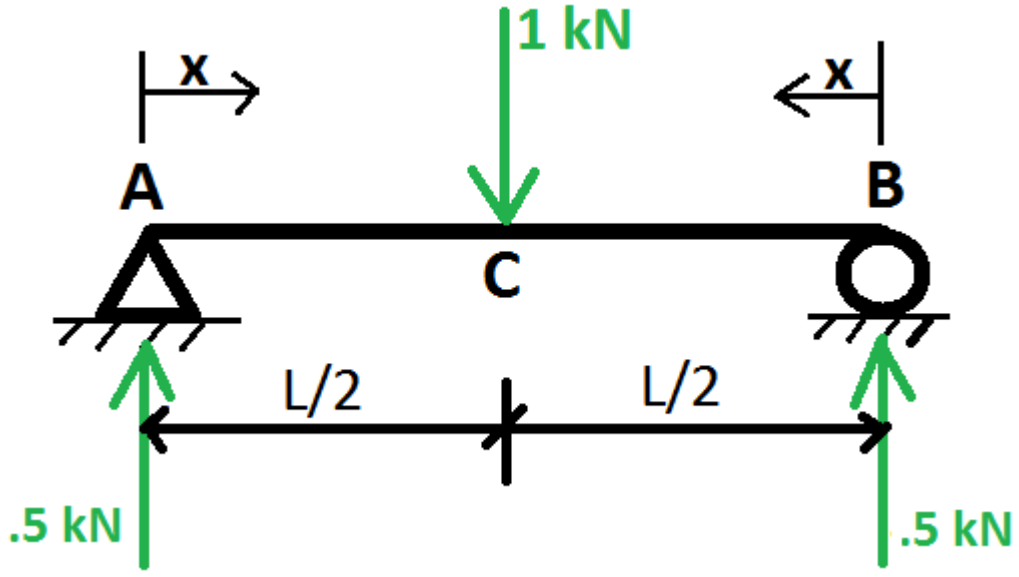
L = beam span

Δ_c = deflection at point C

The diagram for M_0 is as follows-



The diagram for M_1 is as follows-



We know that $\Delta=\int \frac{M_0\, M_1}{EI}\, dx$

Hence,

$$EI\Delta_c=\int_0^{L/2}{}^{\text{A}\to\text{C}} \left\{\frac{P}{2}.x\right\}\left\{\frac{1}{2}.x\right\} dx + \int_0^{L/2}{}^{\text{B}\to\text{C}} \left\{\frac{P}{2}.x\right\}\left\{\frac{1}{2}.x\right\} dx$$

$$= \int_0^{L/2} \frac{P}{4} x^2 dx + \int_0^{L/2} \frac{P}{4} x^2 dx$$

$$= \frac{P}{4}\left[\frac{x^3}{3}\right]_0^{L/2} + \frac{P}{4}\left[\frac{x^3}{3}\right]_0^{L/2}$$

$$= 2\left(\frac{PL^3}{96}\right)$$

$$= \frac{PL^3}{48}$$

$\Delta_c = \frac{PL^3}{48\, EI}$ (Answer)

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Checked by: Dr. Latifee, December 7, 2015