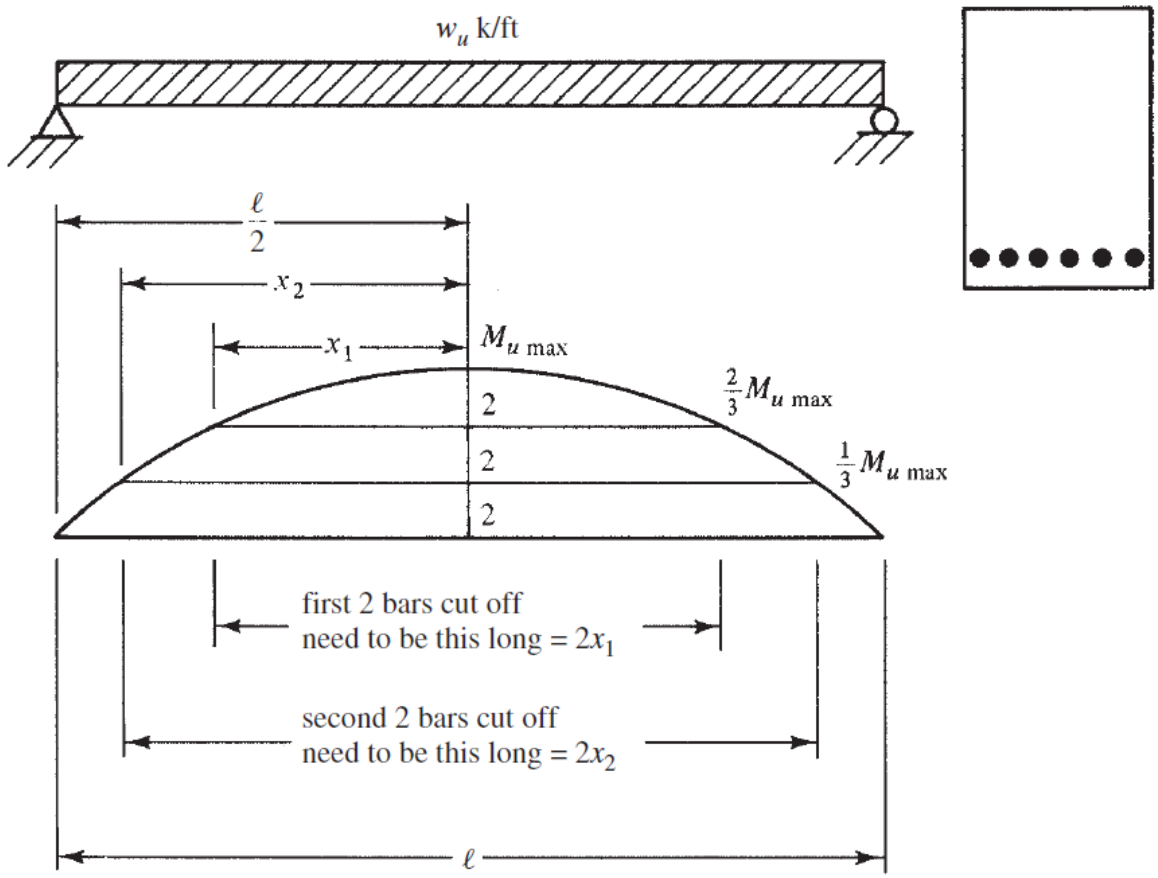
Bond, Development Lengths, and Hook



Theoretical cutoff locations for a simple span beam.

The steel and the concrete should stick together, or *bond*, so that they will act as a unit.

Even if the bars are completely separated from the concrete over considerable parts of their length, the ultimate strength of the beam will not be affected if the bars are so anchored at their ends that they cannot pull loose.

Q. What factors contribute to bonding of rebar to the concrete.

Answer: The bonding of the reinforcing bars to the concrete is due to several factors, including

1. the chemical adhesion between the two materials,
2. the friction due to the natural roughness of the bars, and
3. the bearing of the closely spaced rib-shaped deformations on the bar surfaces against the concrete.

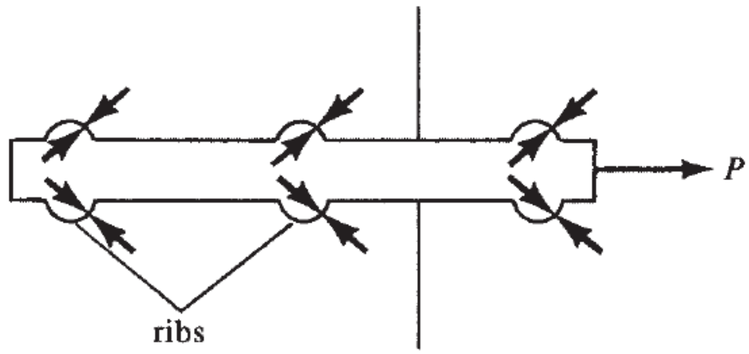
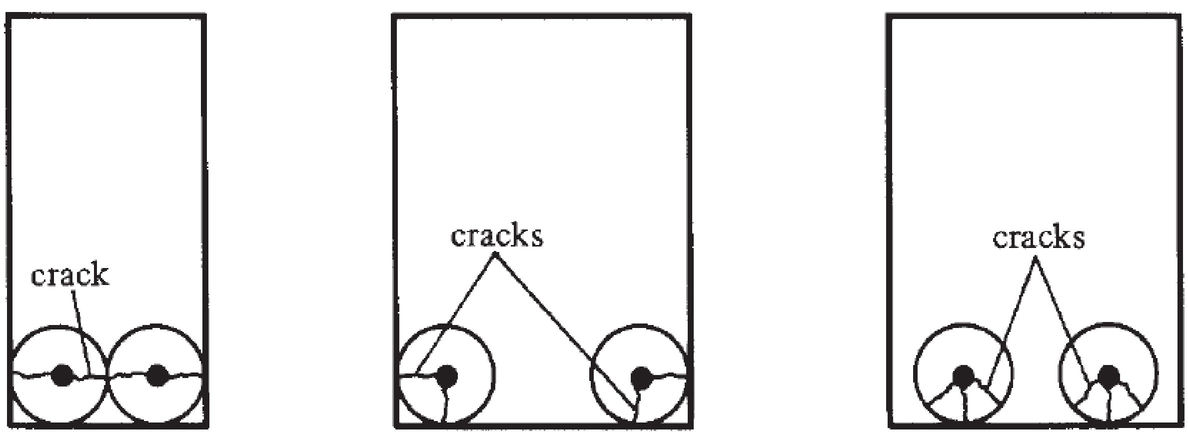


Figure: Bearing forces on bar and bearing of bar ribs on concrete.



(a) Side cover and one-half clear spacing between bars < bottom cover

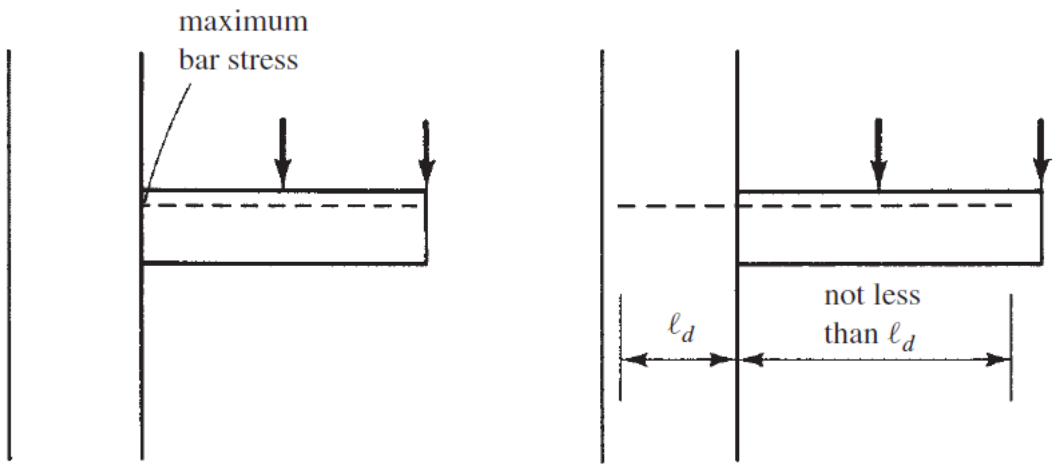
(b) Cover on sides and bottom equal and < one-half clear spacing between bars

(c) Bottom cover < side cover and < one-half clear spacing between bars

Figure: Types of bond failures.

Consider a cantilever beam. It can be seen that both the maximum moment in the beam and the maximum stresses in the tensile bars occur at the face of the support. Theoretically, a small distance back into the support the moment is zero, and thus it would seem that reinforcing bars would no longer be required. If the bars were stopped at the face of the support, the beam would fail.

The bar stresses must be transferred to the concrete by bond between the steel and the concrete before the bars can be cut off. In this case the bars must be extended some distance back into the support and out into the beam to anchor them or develop their strength. This distance, called the development length (*l*d), is shown in Figure.



(a) No development length at support (beam will fail)

(b) Bars extended into the support a distance = ℓ_d

Figure: Development length in a cantilever support.

Development Length:

It can be defined as the minimum length of embedment of bars that is necessary to permit them to be stressed to their yield point plus some extra distance to ensure member toughness. A similar case can be made for bars in other situations and in other types of beams.

The design is based on the reinforcement attaining the yield stress, so the reinforcement must be properly bonded to the concrete for a finite length in order not to slip. This sufficient length to anchor bars near the end of connections is referred to as **the development length**, *l*d.

The development lengths used for deformed bars or wires in tension may not be less than the values computed with ACI Equation 12-1 or 12 in. If the equation is written as (ld /db), the results obtained will be in terms of bar diameters. This form of answer is very convenient to use as, say, 30 bar diameters, 40 bar diameters, and so on.

$$\ell_d = \frac{3}{40}\frac{f_y}{\lambda\sqrt{f'_c}}\frac{\psi_t\psi_e\psi_s}{\left(\frac{c_b + K_{tr}}{d_b}\right)}d_b \qquad \text{(ACI Equation 12-1)}$$

or

$$\frac{\ell_d}{d_b} = \frac{3}{40}\frac{f_y}{\lambda\sqrt{f'_c}}\frac{\psi_t\psi_e\psi_s}{\left(\frac{c_b + K_{tr}}{d_b}\right)}$$

Or in SI units,

$$\ell_d = \frac{9}{10}\frac{f_y}{\lambda\sqrt{f'_c}}\frac{\psi_t\psi_e\psi_s}{\left(\frac{c_b + K_{tr}}{d_b}\right)}d_b$$

(1) ψ_t = *reinforcement location factor*

Horizontal reinforcement so placed that more than 12 in. of fresh concrete is cast in the member below the development length or splice 1.3

Other reinforcement 1.0

(2) ψ_e = *coating factor*

Epoxy-coated bars or wires with cover less than $3d_b$, or clear spacing less than $6d_b$ 1.5

All other epoxy-coated bars or wires 1.2

Uncoated and zinc-coated reinforcement 1.0

However, the product of $\psi_t\psi_e$ need not be taken as greater than 1.7.

(3) ψ_s = *reinforcement size factor*

No. 6 and smaller bars and deformed wires 0.8

No. 7 and larger bars 1.0

In SI units

No. 19 and smaller bars and deformed wires 0.8

No. 22 and larger bars 1.0

(4) λ (*lambda*) = *lightweight aggregate concrete factor*

When lightweight aggregate concrete is used, λ shall not exceed 0.75

However, when f_{ct} is specified, λ shall be permitted to be taken as $6.7\sqrt{f'_c}/f_{ct}$

It's $\sqrt{f'_c}/1.8f_{ct}$ in SI.

but not greater than 1.0

When normal weight concrete is used 1.0

(5) c_b = *spacing or cover dimension, in.*

Use the smaller of either the distance from the center of the bar or wire to the nearest concrete surface, or one-half the center-to-center spacing of the bars or wires being developed.

In this expression, K_{tr} is a factor called the *transverse reinforcement index*. It is used to account for the contribution of confining reinforcing (stirrups or ties) across possible splitting planes.

$$K_{tr} = \frac{40A_{tr}}{sn}$$

where

A_{tr} = the total cross-sectional area of all transverse reinforcement having the center-to-center spacing s and a yield strength f_{yt}

n = the number of bars or wires being developed along the plane of splitting. If steel is in two layers, n is the largest number of bars in a single layer.

s = center-to-center spacing of transverse reinforcing

The code in Section 12.2.3 conservatively permits the use of $K_{tr} = 0$ to simplify the calculations, even if transverse reinforcing is present. ACI 12.2.3 limits the value of $(c_b + K_{tr})/d_b$ used in the equation to a maximum value of 2.5. (It has been found that if values larger than 2.5 are used, the shorter development lengths resulting will increase the danger of pullout-type failures.)

Example:

Determine the development length required for the #8 uncoated bottom bars shown in Figure

(a) assume $K_{tr} = 0$ and

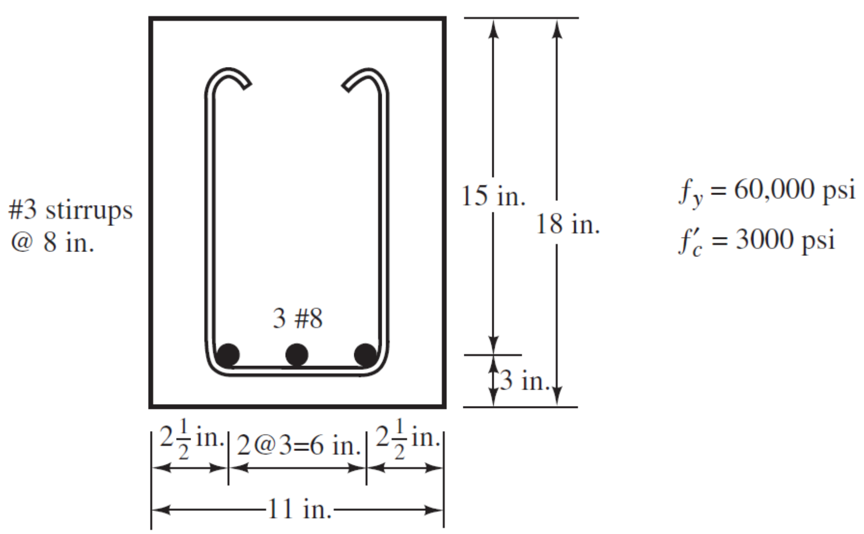
(b) use the computed value of K_{tr}.

SOLUTION

From Table 7.1

$\psi_t = 1.0$ for bottom bars

$\psi_e = 1.0$ for uncoated bars



$\psi_s = 1.0$ for #8 bars
$\lambda = 1.0$ for normal-weight concrete
$c_b =$ side cover of bars measured from center of bars $= 2\frac{1}{2}$ in.

or

$c_b =$ one-half of c. to c. spacing of bars $= 1\frac{1}{2}$ in. ←

(a) Using ACI Equation 12-1 with $K_{tr} = 0$

$$\frac{c_b + K_{tr}}{d_b} = \frac{1.50 \text{ in.} + 0 \text{ in.}}{1.00 \text{ in.}} = 1.50 < 2.50 \quad \underline{\underline{\text{OK}}}$$

$$\frac{\ell_d}{d_b} = \frac{3}{40} \frac{f_y}{\lambda\sqrt{f'_c}} \frac{\psi_t \psi_e \psi_s}{\left(\dfrac{c_b + K_{tr}}{d_b}\right)}$$

$$= \left(\frac{3}{40}\right)\left[\frac{60{,}000 \text{ psi}}{(1.0)\sqrt{3000} \text{ psi}}\right]\frac{(1.0)(1.0)(1.0)}{1.50} = \underline{\underline{55 \text{ diameters}}}$$

(b) Using Computed Value of K_{tr} and ACI Equation 12-1

$$K_{tr} = \frac{40A_{tr}}{sn} = \frac{(40)(2)(0.11 \text{ in.}^2)}{(8 \text{ in.})(3)} = 0.367 \text{ in.}$$

$$\frac{c_b + K_{tr}}{d_b} = \frac{1.50 \text{ in.} + 0.367 \text{ in.}}{1.0 \text{ in.}} = 1.867 < 2.5 \quad \underline{\underline{\text{OK}}}$$

$$\frac{\ell_d}{d_b} = \left(\frac{3}{40}\right)\left(\frac{60{,}000 \text{ psi}}{\sqrt{3000} \text{ psi}}\right)\frac{(1.0)(1.0)(1.0)(1.0)}{1.867} = \underline{\underline{44 \text{ diameters}}}$$

Example 2

The #7 bottom bars shown in Figure 7.9 are epoxy coated. Assuming normal-weight concrete, $f_y = 60{,}000$ psi, and $f'_c = 3500$ psi, determine required development lengths

(a) Using the simplified equations of Table 7.2.
(b) Using the full ACI Equation 12-1 with the calculated value of K_{tr}.
(c) Using ACI Equation 12-1 with $K_{tr} = 0$.

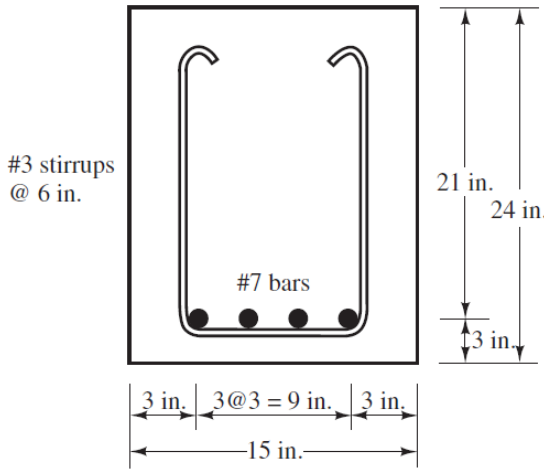
SOLUTION

With reference to Table 7.1

$\psi_t = 1.0$ for bottom bars
$\psi_e = 1.5$ for epoxy-coated bars with clear spacing $< 6d_b$
$\psi_t \psi_e = (1.0)(1.5) = 1.5 < 1.7$ $\underline{\underline{\text{OK}}}$
$\psi_s = 1.0$ for #7 and larger bars
$\lambda = 1.0$ for normal-weight concrete
$c_b =$ cover $= 3$ in.

or

$c_b =$ one-half of c. to c. spacing of bars $= 1\frac{1}{2}$ in. ← controls



(a) Using Simplified Equation

$$\frac{\ell_d}{d_b} = \frac{f_y \psi_t \psi_e}{20\lambda\sqrt{f'_c}} = \frac{(60{,}000 \text{ psi})(1.0)(1.5)}{20(1.0)\sqrt{3500} \text{ psi}} = 76 \text{ diameters}$$

(b) Using ACI Equation 12-1 with Computed Value of K_{tr}

$$K_{tr} = \frac{40A_{tr}}{sn} = \frac{(40)(2)(0.11 \text{ in.}^2)}{(6 \text{ in.})(4)} = 0.367 \text{ in.}$$

$$\frac{c_b + K_{tr}}{d_b} = \frac{1.5 \text{ in.} + 0.367 \text{ in.}}{0.875 \text{ in.}} = 2.13 < 2.50 \quad \underline{\underline{\text{OK}}}$$

$$\frac{\ell_d}{d_b} = \frac{3}{40}\frac{f_y}{\lambda\sqrt{f'_c}}\frac{\psi_t\psi_e\psi_s}{\frac{c_b + K_{tr}}{d_b}}$$

$$= \left(\frac{3}{40}\right)\left(\frac{60{,}000 \text{ psi}}{(1.0)\sqrt{3500} \text{ psi}}\right)\frac{(1.0)(1.5)(1.0)}{2.13}$$

$$= \underline{\underline{54 \text{ diameters}}}$$

(c) Using ACI Equation 12-1 with $K_{tr} = 0$

$$\frac{c_b + K_{tr}}{d_b} = \frac{1.5 \text{ in.} + 0 \text{ in.}}{0.875 \text{ in.}} = 1.71 < 2.50 \quad \underline{\underline{\text{OK}}}$$

$$\frac{\ell_d}{d_b} = \left(\frac{3}{40}\right)\left(\frac{60{,}000 \text{ psi}}{(1.0)\sqrt{3500} \text{ psi}}\right)\frac{(1.0)(1.5)(1.0)}{1.71}$$

$$= \underline{\underline{67 \text{ diameters}}}$$

Hooks

When sufficient space is not available to anchor tension bars by running them straight for their required development lengths hooks may be used. (Hooks are considered ineffective for compression bars for development length purposes.)

Development Length in Compression

$$l_d = \frac{0.02 d_b F_y}{\sqrt{f_c'}} \leq 0.0003 d_b F_y$$

Hook Bends and Extensions

The minimum hook length is:

$$l_{dh} = \frac{1200 d_b}{\sqrt{f_c'}}$$

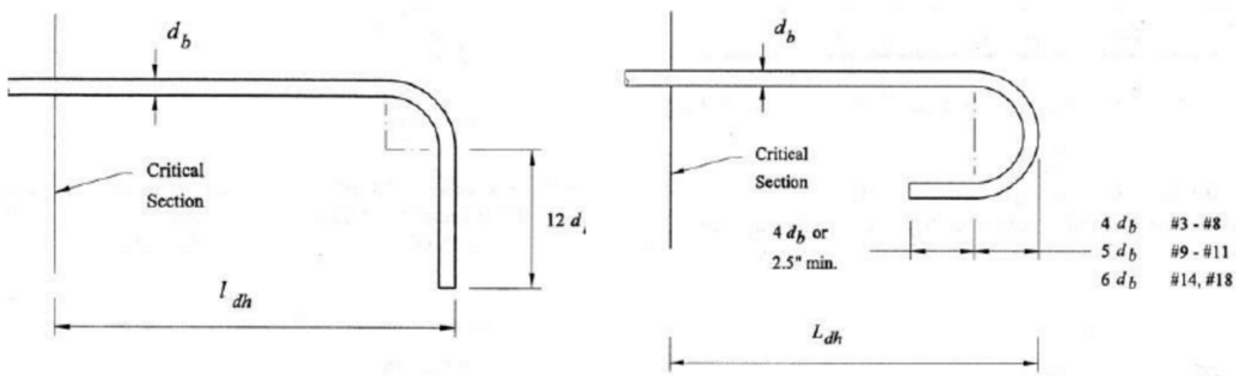
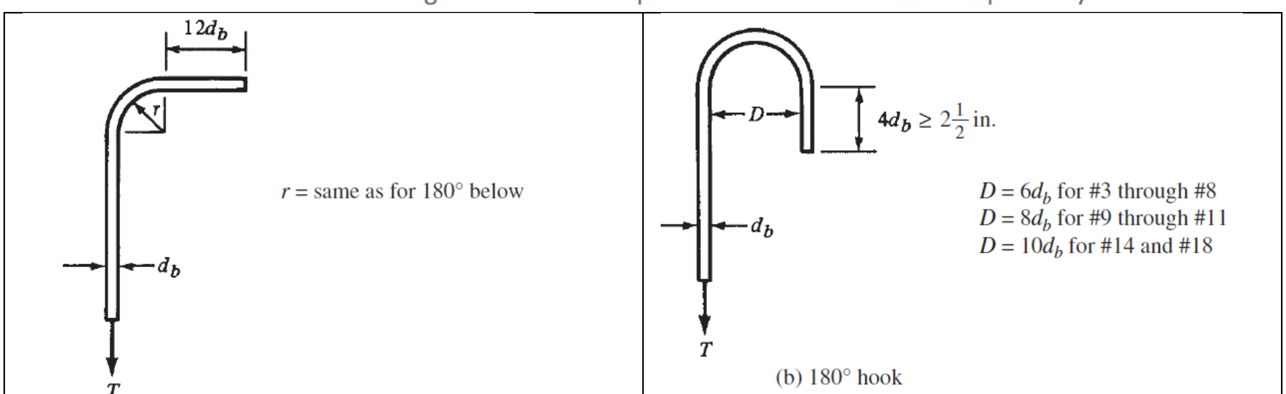


Figure: Minimum requirements for 90° and 180° respectively.

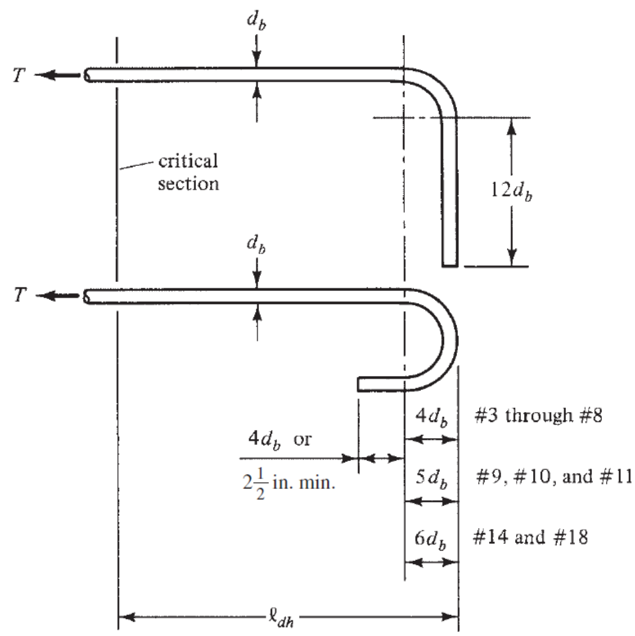


(a) 90° hook

(b) 180° hook

Question: Show Details of the standard 90◦ and 180◦ hooks.

Details of the standard 90◦ and 180◦ hooks specified in Sections 7.1 and 7.2 of the ACI Code. Either the 90◦ hook with an extension of 12 bar diameters (12db) at the free end or the 180◦ hook with an extension of 4 bar diameters (4db) but not less than 2.5 in. may be used at the free end. The radii and diameters shown are measured on the inside of the bends.



Example:

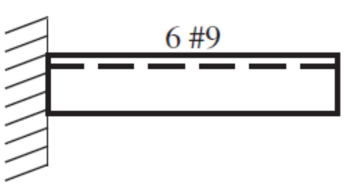
Determine the development or embedment length required for the epoxy-coated bars of the beam shown in Figure

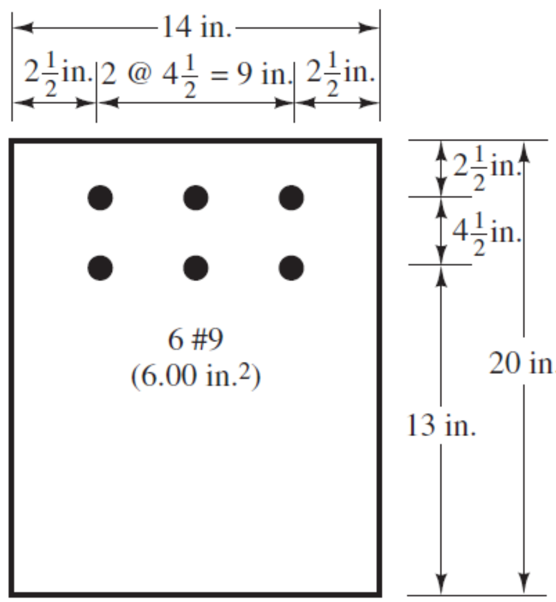
(a) If the bars are straight, assuming $K_{tr} = 0$.

(b) If a 180° hook is used.

(c) If a 90° hook is used.

The six #9 bars shown are considered to be top bars. $f'_c = 4000$ psi and $f_y = 60{,}000$ psi.





SOLUTION

(a) Straight Bars

$$
\begin{aligned}
\psi_t &= 1.3 \text{ for top bars} \\
\psi_e &= 1.5 \text{ for coated bars with cover} < 3d_b \text{ or clear spacing} < 6d_b \\
\psi_t\psi_e &= (1.3)(1.5) = 1.95 > 1.7 \quad \underline{\therefore \text{ Use } 1.7} \\
\psi_s &= 1.0 \text{ for 9 bars} \\
\lambda &= 1.0 \text{ for normal-weight concrete} \\
c_b &= \text{side cover} = \text{top cover} = 2.5 \text{ in.} \\
c_b &= \text{one-half of c. to c. spacing of bars} = 2.25 \text{ in.} \leftarrow \text{controls}
\end{aligned}
$$

$$\frac{c_b + K_{tr}}{d_b} = \frac{2.25 \text{ in.} + 0 \text{ in.}}{1.128 \text{ in.}} = 1.99 < 2.5 \quad \underline{\underline{\text{OK}}}$$

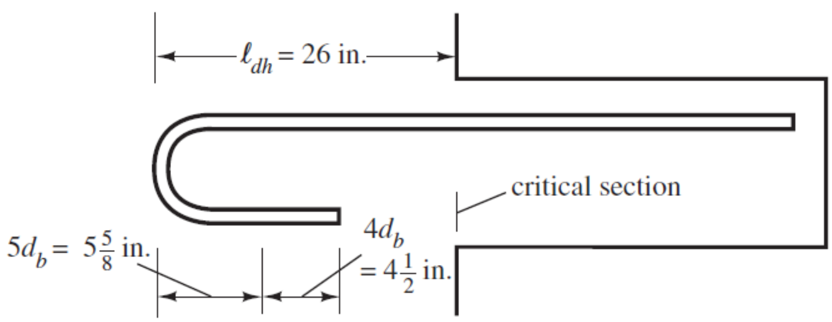
$$\frac{\ell_d}{d_b} = \left(\frac{3}{40}\right)\left(\frac{60{,}000 \text{ psi}}{(1.0)\sqrt{4000} \text{ psi}}\right)\frac{(1.7)(1.0)}{1.99} = 61 \text{ diameters}$$

$$\ell_d = (61)(1.128 \text{ in.}) = \underline{\underline{69 \text{ in.}}}$$

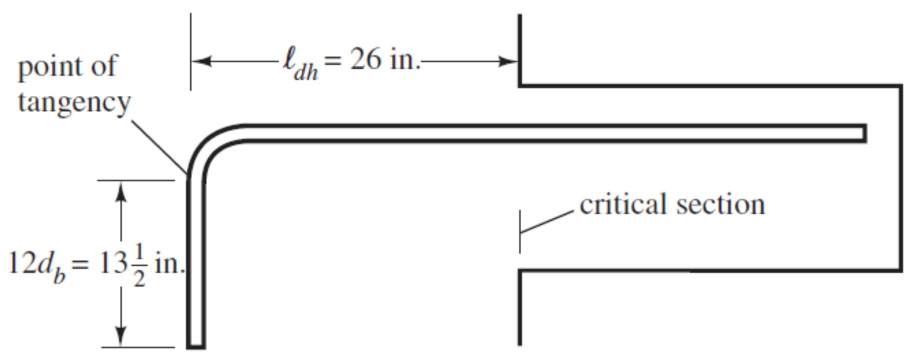
(b) Using 180◦ hooks, note that ψe = 1.2 as required in ACI Section 12.5.2 for epoxy-coated hooks

$$\ell_{dh} = \frac{0.02\psi_e f_y d_b}{\lambda\sqrt{f'_c}} = \frac{(0.02)(1.2)(60{,}000 \text{ psi})(1.128 \text{ in.})}{(1.0)\sqrt{4000} \text{ psi}}$$

$$= 25.68 \text{ in.} \qquad \underline{\underline{\text{Say 26 in.}}}$$



(c) Using 90◦ hooks, l*dh* = 26 in. as the 0.8 reduction factor does not apply because ties or stirrups are not provided.



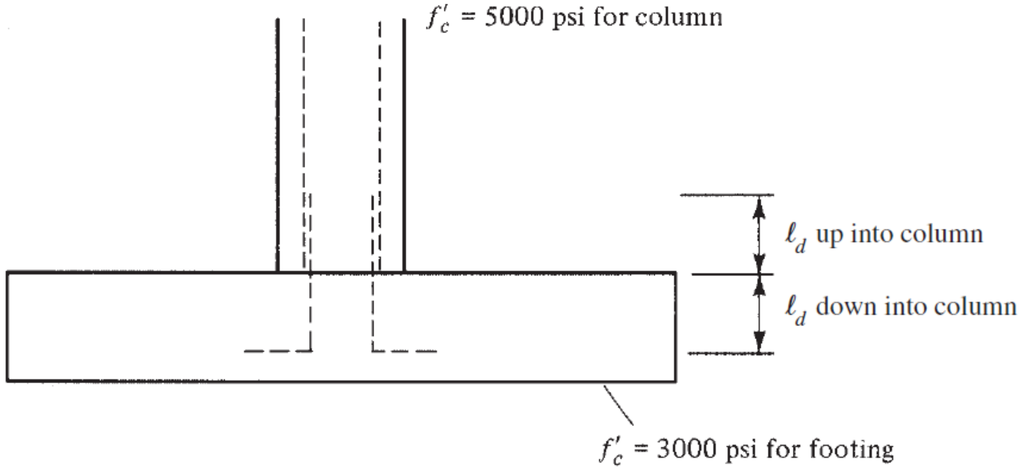
Development Lengths for Compression Bars

$$\ell_{dc} = \frac{0.02 f_y d_b}{\lambda\sqrt{f'_c}} \geq 0.0003\ f_y d_b \text{ but not less than 8 in.}$$

Or in SI units

$$\ell_{dc} = \frac{0.02 f_y d_b}{\lambda\sqrt{f'_c}} \geq 0.0003\ f_y d_b \text{ but not less than 200 mm}$$

Example: The forces in the column bars of Figure are to be transferred into the footing with #9 dowels. Determine the development lengths needed for the dowels (a) down into the footing and (b) up into the column if fy = 60,000 psi. The concrete in both the column and the footing is normal weight.



(a) Down into the footing,

$$\ell_{dc} = \frac{0.02 d_b f_y}{\lambda\sqrt{f'_c}} = \frac{(0.02)(1.128 \text{ in.})(60{,}000 \text{ psi})}{(1.0)\sqrt{3000} \text{ psi}} = 24.71 \text{ in.} \leftarrow$$

$$\ell_{dc} = (0.0003)(1.128 \text{ in.})(60{,}000 \text{ psi}) = 20.30 \text{ in.}$$

Hence $\ell_d = 24.71$ in., say 25 in., as there are no applicable modification factors. Under no circumstances may ℓ_d be less than 8 in.

(b) Up into column,

$$\ell_{dc} = \frac{(0.02)(1.128 \text{ in.})(60{,}000 \text{ psi})}{(1.0)\sqrt{5000} \text{ psi}} = 19.14 \text{ in.}$$

$$\ell_{dc} = (0.0003)(1.128 \text{ in.})(60{,}000 \text{ psi}) = 20.30 \text{ in.} \leftarrow$$

Hence $\ell_d = 20.30$ in., say 21 in. (can't be $<$ 8 in.), as there are no applicable modification factors. (*Answer:* Extend the dowels 25 in. down into the footing and 21 in. up into the column.)