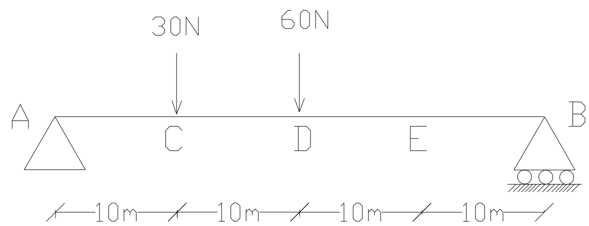
Question: Find the deflections at D, C and E by Unit Load/Virtual Work method for the following simply supported beam.



Solution: The formula of deflection according to unit load method

Deflection, $\Delta = \int \frac{M_0 M_1}{EI} dx$

Where , M_0 = Bending moment distribution due to actual or real loading

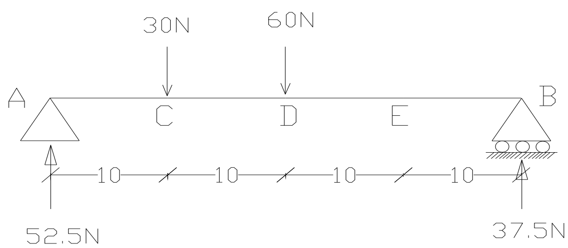
M_1= Bending moment distribution due to virtual or unit loading

E = Modulus off elasticity of the material of beam

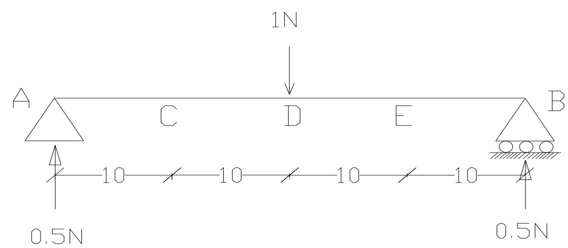
I = Moment of inertia of beam section

Δ = Deflection.

Determination of Δ_D :



M_o diagram



M_1 diagram

$$EI.\Delta_D = \int_0^{10} {}^{A\to C}\{52.5 * x\}\{.5 * x\}\,dx + \int_0^{10} {}^{C\to D}\{52.5 * (10 + x) - 30 * x\}\{.5 * (10 + x)\}\,dx$$

$$+\int_0^{10} {}^{E\to D}\{37.5 * (x + 10\}\{.5 * (x + 10)\}\,dx + \int_0^{10} {}^{B\to E}\{37.5 * x\}\{.5 * x\}\,dx$$

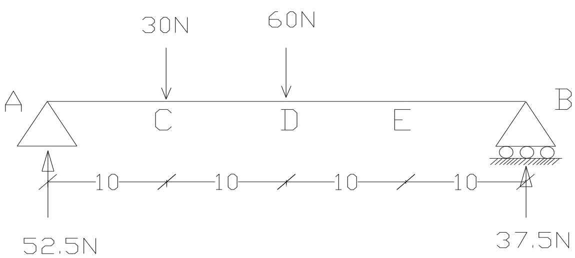
=8750+48750+43750+6250

=107500

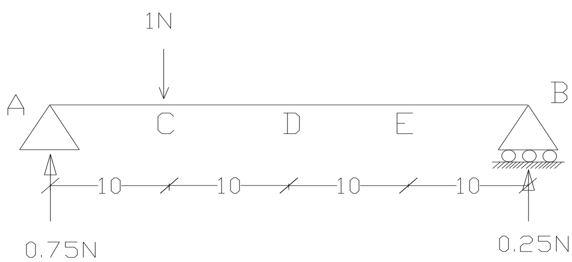
$\Delta_D = \frac{107500}{EI}$ N.m^3

$\Delta_D = \frac{107.5}{EI}*10^3$ N.m^3

Determination of Δ_c :



M_0 diagram



M_1 diagram

EI.Δ_c = $\int_0^{10} {}^{A\rightarrow C}\{52.5 * x\}\{.75 * x\}\,dx + \int_0^{10} {}^{B\rightarrow E}\{37.5 * x\}\{.25 * x\}\,dx$

$+\int_0^{10} {}^{E\rightarrow D}\{37.5 * (x+10)\}\{.25 * (x+10)\}\,dx + \int_0^{10} {}^{D\rightarrow C}\{37.5 * (x+20) - 60 * x\}\{.25 * (x+20)\}\,dx$

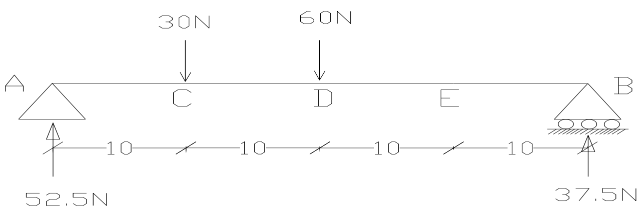
= 13125+3125+21875+39375

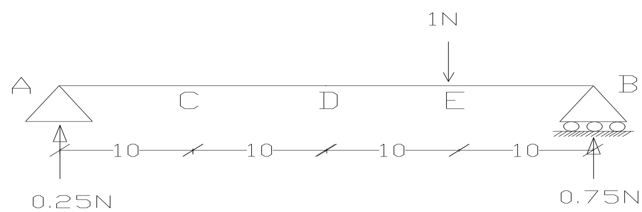
= 77500

$\Delta_C = \frac{77500}{EI}$ N.m^3

$\Delta_C = \frac{77.5}{EI} * 10^3$ N.m^3

Determination of Δ_E :





M_0 diagram

M_1 diagram

$$EI.\Delta_E = \int_0^{10} {}^{B\to E}\{37.5 * x\}\{.75 * x\}\,dx + \int_0^{10} {}^{A\to C}\{52.5 * x\}\{.25 * x\}\,dx$$

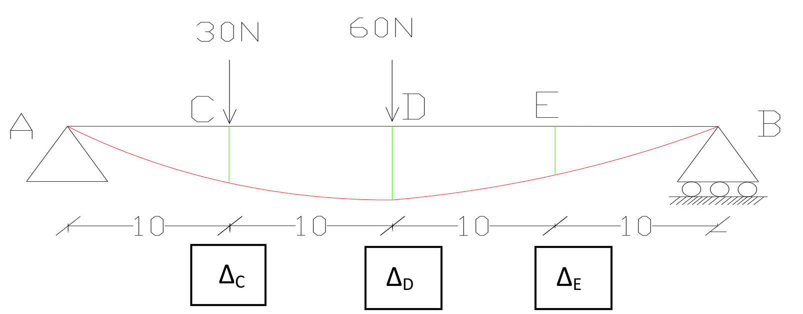
$$+\int_0^{10} {}^{C\to D}\{52.5 * (x+10)\}\{.25 * (x+10)\}\,dx$$

$$+\int_0^{10} {}^{D\to E}\{52.5 * (x+20) - 30 * (x+10) - 60 * x\}\{.25 * (x+20)\}\,dx$$

= 9375+4375+24375+34375 = 72500

$$\Delta_E = \frac{72500}{EI}\ N.m^3$$

$$\Delta_E = \frac{72.5}{EI} * 10^3\ N.m^3$$



Done by: Mr. Wakil Ahmed; **Checked by: Dr. Latifee, December 7, 2015**