Analysis of Doubly reinforced rectangular Beam

Example: Doubly Reinforced Section

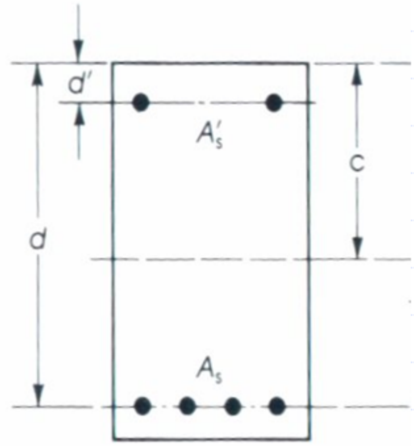
Given:

$f'_c = 4000$ psi $f_y = 60$ ksi

$A'_s = 2$ #5 $A_s = 4$ #7

$d' = 2.5$ in. $d = 15.5$ in

$h = 18$ in. $b = 12$ in.



Calculate M_n for the section for the given compression steel.

Compute the reinforcement coefficients, the area of the bars #7 (0.6 in^2) and #5 (0.31 in^2)

$$A_s = 4\left(0.6 \text{ in}^2\right) = 2.4 \text{ in}^2$$

$$A'_s = 2\left(0.31 \text{ in}^2\right) = 0.62 \text{ in}^2$$

$$\rho = \frac{A_s}{bd} = \frac{2.4 \text{ in}^2}{(12 \text{ in.})(15.5 \text{ in.})} = 0.0129$$

$$\rho' = \frac{A'_s}{bd} = \frac{0.62 \text{ in}^2}{(12 \text{ in.})(15.5 \text{ in.})} = 0.0033$$

Compute the effective reinforcement ratio and minimum ρ

$$\rho_{eff} = \rho - \rho' = 0.0129 - 0.0033 = 0.00957$$

$$\rho = \frac{200}{f_y} = \frac{200}{60000} = 0.00333$$

$$\text{or } \frac{3\sqrt{f_c}}{f_y} = \frac{3\sqrt{4000}}{60000} = 0.00316$$

$$\rho \geq \rho_{min} \Rightarrow 0.0129 \geq 0.00333 \text{ OK!}$$

Compute the effective reinforcement ratio and minimum ρ

$$(\rho - \rho') \geq \left(\frac{\beta_1 (0.85 f'_c) d'}{d\, f_y}\right)\left(\frac{87}{87 - f_y}\right)$$

$$\geq \left(\frac{0.85(0.85(4\text{ ksi}))(2.5\text{ in.})}{60\text{ ksi}(15.5\text{ in.})}\right)\left(\frac{87}{87-60}\right) = 0.0398$$

$0.00957 \not\geq 0.0398$ Compression steel has not yielded.

Instead of iterating the equation use the quadratic method

$$0.85 f_c b \beta_1 c^2 + \left(A_s' E_s \varepsilon_{cu} - A_s f_y \right) c - A_s' E_s \varepsilon_{cu} d' = 0$$

$$0.85(4 \text{ ksi})(12 \text{ in.})(0.85) c^2 +$$

$$+\left[\left(\left(0.62 \text{ in}^2\right)(29000 \text{ ksi})(0.003) - \left(2.4 \text{ in}^2\right)(60 \text{ ksi})\right)\right] c$$

$$-\left(0.62 \text{ in}^2\right)(29000 \text{ ksi})(0.003)(2.5 \text{ in.}) = 0$$

$$34.68 c^2 - 90.06 c - 134.85 = 0$$

$$c^2 - 2.5969 c - 3.8884 = 0$$

Solve using the quadratic formula

$$c^2 - 2.5969 c - 3.8884 = 0$$

$$c = \frac{2.5969 \pm \sqrt{(-2.5969)^2 - 4(-3.8884)}}{2}$$

$$c = 3.6595 \text{ in.}$$

Find the f_s'

$$f_s' = \left(1 - \frac{d'}{c}\right) E_s \varepsilon_{cu} = \left(1 - \frac{2.5 \text{ in.}}{3.659 \text{ in.}}\right) 87 \text{ ksi}$$

$$= 27.565 \text{ ksi}$$

Check the tension steel.

$$\varepsilon_s = \left(\frac{15.5 \text{ in.} - 3.659 \text{ in.}}{3.659 \text{ in.}}\right) 0.003 = 0.00971 \geq 0.00207$$

Check to see if c works

$$c = \frac{A_s f_y - A_s' f_s'}{0.85 f_c \beta_1 b} = \frac{\left(2.4 \text{ in}^2\right)\left(60 \text{ ksi}\right) - \left(0.62 \text{ in}^2\right)\left(27.565 \text{ ksi}\right)}{0.85\left(4 \text{ ksi}\right)\left(0.85\right)\left(12 \text{ in.}\right)}$$

$$c = 3.659 \text{ in.}$$

Compute the moment capacity of the beam

$$M_n = \left(A_s f_y - A_s' f_s'\right)\left(d - \frac{a}{2}\right) + A_s' f_s' \left(d - d'\right)$$

$$= \begin{pmatrix} \left(2.4 \text{ in}^2\right)\left(60 \text{ ksi}\right) \\ -\left(0.62 \text{ in}^2\right)\left(27.565 \text{ ksi}\right) \end{pmatrix} \left(15.5 \text{ in.} - \frac{0.85\left(3.659 \text{ in.}\right)}{2}\right)$$

$$+\left(0.62 \text{ in}^2\right)\left(27.565 \text{ ksi}\right)\left(15.5 \text{ in.} - 2.5 \text{ in.}\right)$$

$$= 1991.9 \text{ k-in.} \Rightarrow 166 \text{ k-ft}$$

The resulting ultimate moment is

$$M_u = \phi M_u = 0.9(166 \text{ k - ft})$$
$$= 149.4 \text{ k - ft}$$