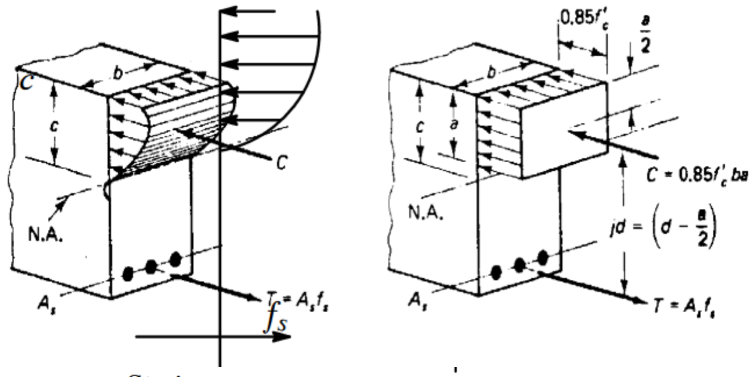
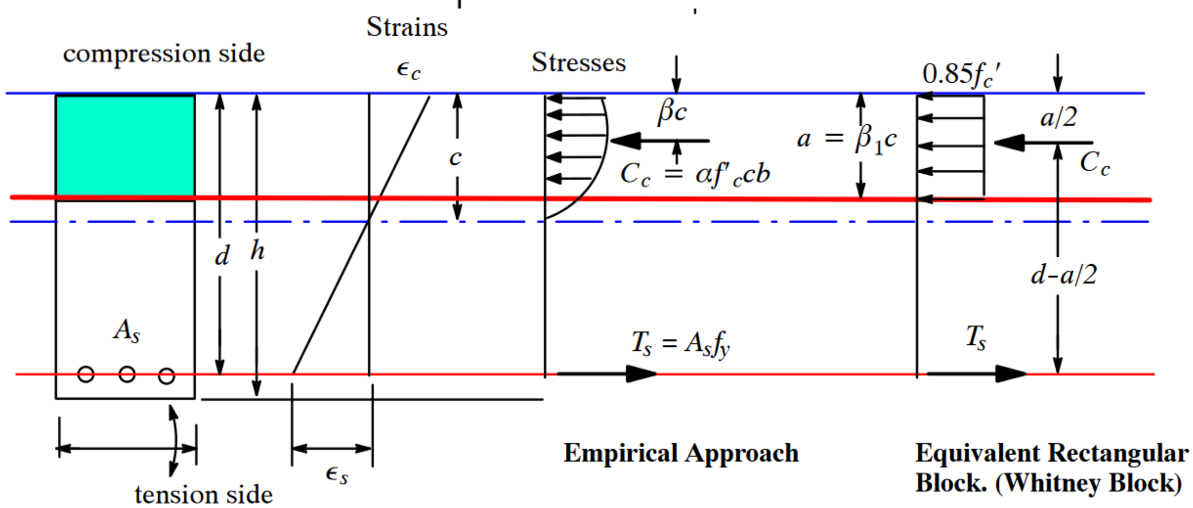
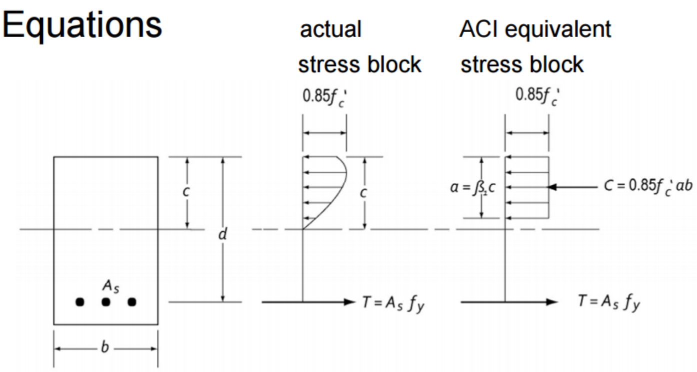
Analysis of a Beam-Finding Nominal Strength or Capacity of a Beam in Flexure by Equivalent Rectangular Stress





Flexure Equations



$$C = T$$

$$0.85f_c'ab = A_s f_y$$

solving for a,

$$a = \frac{A_s f_y}{0.85f_c'b} = \frac{\rho f_y d}{0.85f_c'}$$

$$\rho = \frac{A_s}{bd}$$

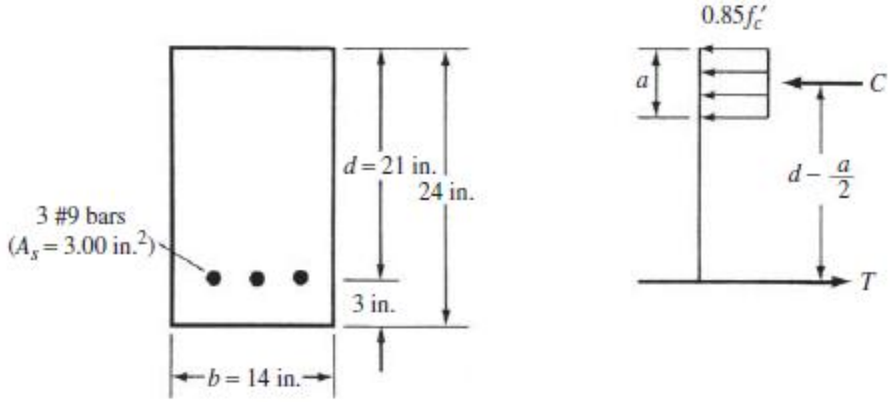
$$M_n = T\left(d - \frac{a}{2}\right) = A_s f_y\left(d - \frac{a}{2}\right)$$

$$M_u = \phi M_n$$

$$M_u = \phi M_n = \phi A_s f_y\left(d - \frac{a}{2}\right)$$

$$M_u = \phi A_s f_y d\left(1 - 0.59\frac{\rho f_y}{f_c'}\right)$$

Example 1 Determine *Mn*, the nominal or theoretical ultimate moment strength of the beam section shown in the Figure below



SOLUTION

Computing Tensile and Compressive Forces *T* and *C*

$$T = A_s f_y = (3.00 \text{ in.}^2)(60 \text{ k/in.}^2) = 180 \text{ k}$$

$$C = 0.85 f'_c ab = (0.85)(3 \text{ k/in.}^2)(a)(14 \text{ in.}) = 35.7a$$

Equating *T* and *C* and Solving for *a*

$$T = C \text{ for equilibrium}$$

$$180 \text{ k} = 35.7a$$

$$a = 5.04 \text{ in.}$$

Computing the Internal Moment Arm and Nominal Moment Capacity

$$d - \frac{a}{2} = 21 \text{ in.} - \frac{5.04 \text{ in.}}{2} = 18.48 \text{ in.}$$

$$M_n = (180 \text{ k})(18.48 \text{ in.}) = 3326.4 \text{ in-k} = \underline{\underline{277.2 \text{ ft-k}}}$$

Example 2 (done by Mr. Naim Hassan, AUST student ID- 13-02-03-048)

A simply supported beam of 20'' total depth & width 12'' , here 72.5 grade steel (500 W) is used and compressive strength of concrete is 3 ksi. 3#9 bar is used Find the moment capacity of the beam. f_y = 60 ksi, f'_c = 3 ksi

SOLUTION: GIVEN

h = 20", d=20"-2.5"=17.5"

Now, T=C

$A_S f_y = 0.85 f'_c a b$

$$a = \frac{A_s f_y}{0.85 f''_c\ b} = \frac{3\times1\times72.5}{0.85\times3\times12}\ ; \quad a = 7.11 \text{ inch}$$

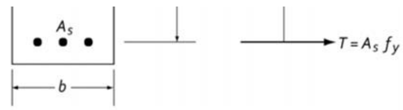
$M_n = T(d-a/2) = A_s f_y (d-a/2)$

$M_n = 3 \times 1 \times 72.5 \times (17.5 - \frac{7.11}{2})$

M_n = 253 k-ft

Example 3

Find a



Find nominal Mn

$$a = \frac{A_s f_y}{.85 f'_c b} = \frac{(2.37)(60000)}{.85(4000)(12)} = 3.49$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right)$$

Find required Mu

Mu = Φ Mn

(using old pre-2005 Φ value)

$$M_u = \phi A_s f_y \left(d - \frac{a}{2}\right)$$

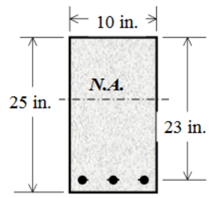
$$M_u = .9(2.37)(60000)\left(17.5 - \frac{3.49}{2}\right)$$

$$M_u = 2017000 \text{ in-lb}$$

$$\underline{M_u = 168 \text{ ft-k}}$$

Example 4

Determine the nominal moment M_n for a beam of cross section shown, where = 4,000 psi. Assume A615 grade 60 steel that has a yield strength of 60 ksi and a modulus of elasticity = 29 × 10⁶ psi.



Area for No. 8 bar = 0.79 in² (see Table 1)

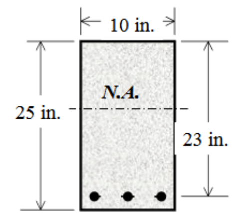
Therefore, $A_s = 3(0.79) = 2.37 \text{ in}^2$ (Also see Table A-2 Text)

Assume that f_y for steel exists subject later check.

$$N_C = N_S$$

$$0.85 f'_c ab = A_s f_y$$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{2.37(60)}{0.85(4)(10)} = 4.18 \text{ in.}$$



Calculation of M_n

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$$M_n = N_C\left(d - \frac{a}{2}\right) = N_T\left(d - \frac{a}{2}\right)$$

$$M_n = 0.85 f'_c ab\left(d - \frac{a}{2}\right) = A_s f_y\left(d - \frac{a}{2}\right)$$

Based on steel:

$$M_n = 2.37(60)\left(23 - \frac{4.18}{2}\right) = 2{,}973.4 \text{ in.-kips}$$

$$= \frac{2{,}973.4}{12} = \boxed{247.8 \text{ ft-kips}}$$